

Peter Gavin Hall 1951–2016

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Peter Hall, in the forty years of his research career, produced work in both probability and statistics, whose breadth and depth must be regarded as phenomenal. He displayed extraordinary technical skills together with remarkable intuition in developing and applying multifaceted mathematical approaches in the whole of his work. The impact of this wide-ranging use of powerful mathematical methods has had a profound effect on much of modern mathematical statistics. After completing his DPhil at Oxford, he remained in Australia for almost all his career although he was renowned as one of the major international figures in probability and statistics. Peter was a mentor to a large group of post-graduate students and post-doctoral colleagues giving encouragement and guidance and he attracted many research visitors contributing greatly to the whole of Australian statistical research. Remarkably, given his immense research output, he took a significant role in both editorial duties in major international journals and in advocacy for mathematics and statistics in Australia. Peter was a man of great charm whose modest demeanour belied his staggering abilities. His loss to mathematics and statistics is great, but is matched by the personal loss to us and to his many friends.

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Early Life and Education

Peter Gavin Hall was born in Sydney, Australia, on 20 November 1951 and, following a diagnosis of acute leukaemia, died in Melbourne on 9 January 2016, while at the height of his intellectual powers. He grew up in Oatley, a southern suburb of Sydney. His father, William Hall, known always as Bill, was a telephone technician. His mother, Ruby Payne-Scott, was a member of a pioneering group of radar scientists during World War 2 and obtained distinction as the first woman radio astronomer.¹ His younger sister, Fiona Hall, is a well-known contemporary artist. In interviews recorded in 2012, and in March and April 2015 (and published posthumously),² Peter gave several insights into his family, his upbringing and his early interests.

Peter attended the local primary school at Oatley West, then went on to a selective state secondary school, Sydney Technical High School, developing an interest in mathematics and physics. He enrolled in a science degree at the University of Sydney in 1970 intending to study physics, but after his first year he chose to take three mathematics subjects, pure mathematics, applied mathematics and mathematical statistics, continuing in his third year to pure mathematics and mathematical statistics and then choosing to take his fourth year honours in mathematical statistics. He was the sole fourth year student in mathematical statistics in that year and it was clear that he was exceptional. Special reading courses in probability and nonparametrics were undertaken with some lecturers, together with courses from Oliver Lancaster FAA. During and after the reading courses it seemed to his lecturers that Peter had a significantly more complete understanding of the course material than they did. He graduated with first class honours and the University Medal.

At the end of 1973, Peter went to the Australian National University (ANU) as a vacation scholar working with Pat (P.A.P.) Moran FRS FAA and Roger Miles. In the acknowledgements of *An Introduction to the Theory of Coverage Processes*, referred to later,



Peter attributed the beginning of his interest in this area to two months spent talking with them about geometric probability.

In March 1974, Peter started a PhD under the supervision of Chris Heyde FAA obtaining, in a short six-month period, several new results on martingales, which are sequences of particular kinds of dependent random variables that are a natural generalization of

sequences of sums of independent random variables. When Peter subsequently decided to accept the offer of a scholarship to Oxford's Brasenose College, Chris Heyde suggested that he write up his results as a master's thesis that would be submitted after the required period. So the PhD enrolment was converted to an enrolment in a Master of Science that was later awarded in 1976, the same year as Peter's DPhil from Oxford.

Peter travelled to Oxford via circuitous means, taking the Trans-Siberian Railway, hard class, fulfilling a long held wish. In Moscow he visited the eminent mathematician Vladimir Zolotarev, who had been a recent visitor to the University of Sydney, and Peter was treated in a most hospitable manner before proceeding on to Oxford. There he was supervised by John Kingman FRS, who recalled, 'to describe him as a research student at that time is unrealistic, because he was already a remarkably independent and mature researcher. He arrived knowing exactly what he wanted to do, and he proceeded to do it'. Further Kingman stated, 'His 1976 DPhil thesis is a model for such dissertations. His highly technical results are clearly explained, their limitations openly admitted, their proofs careful without being pedantic', and noted, 'I saw no signs of an interest in statistics. At Oxford he was a pure probabilist'.³

Private Life

It was while in Oxford in late 1974 that Peter met his future wife, Jeannie Jean Chien Lo. Jeannie had joined the British Hong Kong Civil Service after graduating from the University of Hong Kong majoring in English literature and comparative literature. In October 1974, she was sent by the British Hong Kong Government to attend a nine-month postgraduate course in administrative development based at the University of Oxford. The course was specially designed for young administrative officers from Hong Kong, with studies in management, international relations, comparative government, urbanization and other issues of relevance to Hong Kong. Jeannie was a member of St Hugh's College whilst at Oxford. She left Oxford in July 1975 to return to Hong Kong to resume her career. Soon after her departure, Peter commenced writing to her. They corresponded on an almost daily basis until their marriage. She visited Oxford again in 1976. Peter left Oxford shortly after her visit to return to Australia, but stopped by Hong Kong in July and August 1976 to visit her and meet her parents. They married in April 1977 (see Fig. 1). Jeannie pursued a career in the public service in Australia after their marriage, initially in the Victorian Public Service in Melbourne, and then in the Australian Public Service in Canberra. She held several high profile positions in Canberra, including Deputy Official Secretary to the Governor-General, Parliamentary Liaison Officer for the House of Representatives and Senior Adviser of the Cabinet Secretariat in the Department of the Prime Minister and Cabinet. So both their lives were taken up by their careers. In particular, Peter's prodigious work ethic and his passion for his research must have required a calm privacy at home. They lived a quiet life depending on one another. Their entertaining was occasional dinner parties for close friends. One exception, well remembered by their visitors, was their hosting barbeques in Tidbinbilla Nature Reserve just outside Canberra for students and visitors. Peter travelled extensively and they kept in constant contact during his absences through telephone calls and later the internet.



Figure 1. Peter and Jeannie in Hong Kong 1978.

From his early years, Peter was interested in steam trains and in photography, maintaining these interests throughout his life. In his private, as in his professional life, he strove for perfection and mastery. His camera was an essential part of his many travels and photographs of steam trains, abandoned farm buildings and countryside around Canberra, and stunning panoramic views of the Scottish highlands, are but a few of his huge photographic collection from around the world (see Fig. 2). Some of his photographs were published in magazines or online photography websites.

Aviation history and aircraft design were other topics on which he could be relied for encyclopaedic knowledge. An avid reader, Peter consumed history and current affairs, and, as can be attested by his lunch companions from ANU, was always able to converse on any topic. Jeannie informs us of his love of poetry, and recalls his reciting 'Said Hanrahan' by John O'Brien, reflecting perhaps his feelings for Australia and the understated humour familiar to his acquaintance. He loved cats from the time of his childhood and, with Jeannie, adopted them from animal shelters (see Fig. 3). Perhaps contrarily, he also loved the birdlife of Canberra and could be found feeding cockatoos, rosellas and galahs daily from his ANU office window.

Peter was an outstanding speaker with an ability to explain concepts of extreme difficulty and complexity in a manner that allowed his audience to appreciate the central ideas. Perhaps his complete mastery of the subject matter permitted this. Most do not know and would take it as a tribute to his determination that, in a matter of weeks, with the help of a speech therapist during his first period in Melbourne, he overcame a minor stutter present since childhood.

In his personal relationships Peter was charming, self-effacing and generous. While he was confident in his mathematical abilities and insights he was quick to give credit to his colleagues and, in particular, to support and encourage younger researchers. While working long hours and with great concentration in his office, Peter maintained a social presence, taking time to join colleagues, visitors and students for general and always genial conversation at morning tea and at sandwich lunches.



Figure 2. Peter taking panoramic photographs at the viaduct near Tyndrum, Scotland, c. 1990s.



Figure 3. Peter and Jeannie with cats at home 2007.

Career

From Oxford, Peter applied for a position of lecturer at the University of Melbourne in 1976. This was a time of straitened financial circumstances for universities in Australia. He received an offer of a fixed three-year term as lecturer with an assurance from the department that the fixed term appointment, which had been advertised as a permanent position, was 'merely a formality' and that a permanent position would eventuate. Trusting this assurance he took up the appointment in August 1976. In 1977, much to his chagrin, he found that the assurance of permanency for the position was not to be honoured due to the university's budgetary constraints. By the end of 1977 he applied for a lectureship at the ANU, and after agreeing to move some of his research to statistics, accepted the lectureship, taking it up in September 1978. In his only full year in Melbourne, Peter had six papers published or accepted for publication, while having the department's highest teaching load. This presaged the immense capacity for productive work that was to become startlingly apparent in the next few years (see Supplementary Material 1).

Peter's initial appointment at ANU was in the Department of Statistics in the School of General Studies (formally renamed the Faculties after 1979). At that time, ANU had two statistics departments; the department in the School of General Studies was responsible for undergraduate teaching and research while the department in the Institute of Advanced Studies was a research department. Peter carried a full teaching load from his appointment in 1978 until the mid-1980s. He taught a range of undergraduate and masters courses, began to supervise PhD students and steadily increased his research output, building up towards the levels for which he is now famous. He was a very well organized, careful teacher. He used the same approach in his research presentations (see Fig. 4), often inadvertently making them seem deceptively straightforward. As Peter's research flourished, he was promoted to senior lecturer in 1983, reader in 1986 and was awarded a special professorship in 1988, to acknowledge his 'outstanding and internationally recognised talents in research'. This was a very unusual promotion at the time, hard fought for by Chip (C. R.) Heathcote: the ANU annual report for 1988 noted that this was the first such award since 1971.

In 1986, Peter's position changed to a joint appointment in the two departments of statistics. As the two departments were physically located on opposite sides of campus, every six months Peter packed his books and papers into boxes and moved them from one location to the other. In 1989, the Department of Statistics in the Institute of Advanced Studies joined with the two Departments of Mathematics (one from the Faculties and one from the Institute of Advanced Studies) to form the School of Mathematical Sciences. The department became the Statistics Research Section in the School of Mathematical Sciences with Peter as its head, although he was still jointly appointed in the Department of Statistics in the Faculties. In 1991, the head of the Centre for Mathematics and its Applications, Neil Trudinger FRS FAA, arranged for Peter to be seconded from the Department of Statistics in the Faculties to join him full-time in the Faculties component of the Centre for Mathematics and its Applications. This unique status was very important to both Neil and Peter because it meant that they were not required to do any teaching (although Peter did teach a probability course at various times) but they were both treated as members of the Faculties and hence were eligible to apply for Australian Research Council (ARC)

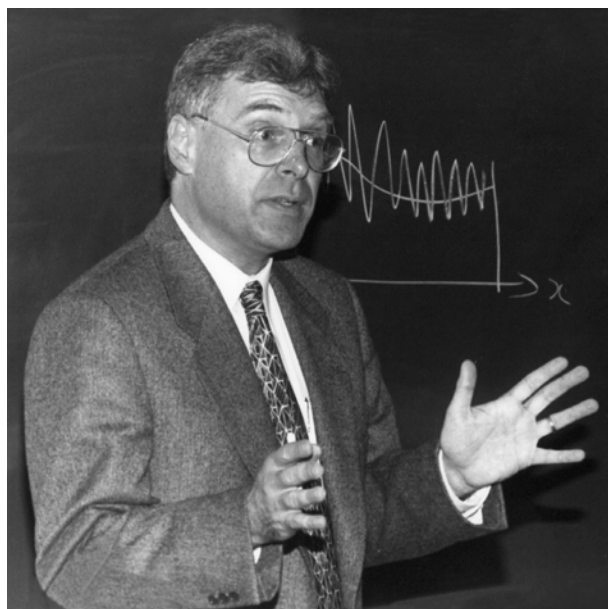


Figure 4. Peter speaking at a seminar believed taken at University of Michigan, Ann Arbor, 1994.

Grants, a privilege not at that time granted to members of the Institute of Advanced Studies. It also allowed considerable freedom to travel and to work at other institutions, such as the Commonwealth Scientific and Industrial Research Organisation (CSIRO) where for some time Peter spent one week per month at its Sydney office. Peter maintained this special status for the rest of his career at ANU. Peter served as acting head of the Centre for Mathematics and its Applications from July 1995 to June 1996 and as acting Dean of the School of Mathematical Sciences in 2001, just before it was renamed the Mathematical Sciences Institute in 2002.

Peter took sabbatical leave to visit the University of North Carolina at Chapel Hill and the University of Glasgow in 1985 (after volunteering for additional teaching in 1984 to secure the leave), but otherwise opportunities for travel were quite limited. Peter began to apply for grants from the late 1980s and, by the standards of the field, was outstandingly successful. Prior to this time there was little opportunity in Australia for funding research in mathematics or statistics and Peter experienced great difficulty in obtaining funds for travel to conferences. He was greatly frustrated by this situation. He did not record the grants in his curriculum vitae (see Supplementary Material 2), but some information can be extracted from the annual reports of the School of Mathematical Sciences. His first applications were for relatively small amounts of money, often joint with other researchers, largely to fund the purchase of computers. Later, applying on his own, Peter won ARC Large Grants as well as prestigious ARC Special Investigator Awards (1992–4 and renewed for 1995–7) and an Australian Professorial Fellowship (2002–6). Peter was thus able to fund some 30 post-doctoral research associates (starting from 1990), a rich stream of research visitors (from which the whole statistical community benefitted) and some of his own international travel. During this period he supervised 23 doctoral students.

Peter took up a 25% appointment as Distinguished Professor of Statistics at the University of California Davis (UC Davis) in 2005. He spent the spring quarter of each year at UC Davis until he fell ill at Davis in 2014. Every other year, Peter taught two courses, an upper division undergraduate probability course and a special topic graduate course on the bootstrap, a statistical method for approximating the properties of statistics using resampling from the original data, to which Peter made fundamentally important research contributions. Otherwise he enjoyed the interactions with researchers at UC Davis and the opportunities to travel and present his research in the USA.

In November 2006, Peter left the ANU and moved to the University of Melbourne where he commenced a five-year appointment as an ARC Federation Fellow. Prior to the move Peter had been a pivotal Chief Investigator in the ‘ANU node’ of the ARC Centre of Excellence for Mathematics and Statistics of Complex Systems, which was centred at the University of Melbourne, so he already had successful research relationships in Melbourne. Later he was chair of the Scientific Advisory Committee of the Australian Mathematical Sciences Institute, which is also based in Melbourne. At the time of his move, he commented explicitly that he was attracted to the University of Melbourne by what he saw as potentially greater opportunities to supervise ‘wonderful PhD students’. Peter did no undergraduate teaching at the University of Melbourne, but did teach part of a master’s course. He was awarded an ARC Laureate Fellowship, 2012–17 and, after playing a key role in winning the ARC Centre of Excellence for Mathematical and Statistical Frontiers, became the first director in 2014. In his group within the School of Mathematics and Statistics at the University of Melbourne, Peter supervised eight PhD students and ten post-doctoral fellows.

Peter took a leading role in advocacy for mathematics and statistics in education and research within Australia. His work with chemist Ben Selinger in the late 1970s led to an appearance before a Senate Inquiry into Agent Orange. He was inaugural chair of the Australian Mathematical Sciences Institute’s Scientific Advisory Committee, strengthening ties between the Australian and international mathematics communities. He took several leadership positions advocating the importance of the mathematical sciences to Australia, including president of the Australian Mathematical Society (2006–8) and chair of the Steering Committee of the Australian Academy of Science’s Decadal Plan for Mathematical Sciences (published in 2016). He was Physical Sciences secretary and vice-president of the Australian Academy of Science in 2008–12. He served on several international scientific committees taking leadership roles and participated in international reviews of mathematical sciences. Additionally, he was on the editorial boards of many international journals, in particular, the *Annals of Statistics* since 1982, and co-editor 2013–15.

In his professional life Peter was direct, determined and put his views in a forthright manner but with a courtesy that maintained great civility in expression of differing views. This permitted him to be a force in several areas both internationally, in roles in the Institute of Mathematical Statistics, as president, and as editor of *Annals of Statistics*, and nationally, in advocating the importance of mathematics and statistics within the Australian Academy of Science and in the Australian Mathematical Sciences Institute.

Peter’s prodigious research output was internationally recognized; he received a large number of the most prestigious academic awards and honours. He gave special named lectures, was elected fellow of several professional societies and was appointed to special professorships around the world. He was elected a Fellow of the Australian Academy of Science in 1987, a Fellow of the Royal Society of London in 2000, a Corresponding Fellow of the Royal Society of Edinburgh in 2002, a Foreign Associate of the US National Academy of Sciences in 2013 and a Fellow of the Academy of the Social Sciences in Australia in 2015. He received honorary doctorates from the Université catholique de Louvain, the University of Glasgow, the University of Sydney and the Universidad de Cantabria. He was made an Officer of the Order of Australia in 2013.

Research

Peter Hall was foremost a mathematician working jointly in probability theory and mathematical statistics. He maintained throughout his career a view that a rigorous mathematical treatment of statistical methods was required to understand and connect these into a coherent study. His inventions were of mathematical devices to consider many of the statistical questions that arose over the last half century. As each new methodology in statistics arose he was in the forefront of solving inherent mathematical problems and in examining the conditions and limits of the techniques. This often led to extensions and variations required to overcome limitations in the original method.

During the first decade of Peter’s research career much of his work (see the bibliography in Supplementary Material 1) was single author. Over his entire career he wrote four monographs, one co-authored by Chris Heyde, and was joined by some 240 collaborators from all corners of the world in publishing more than 600 papers. In particular, he wrote 32 papers with Aurore Delaigle, 23 with Mike Titterton (see Fig. 5), 21 with Steve Marron, and collaborated with many others ranging from established and well-known researchers to postgraduates. All of them testify both to the speed and insight he brought to the work and to his unflinching courtesy and generosity as a co-author.

In the pre-interview notes for his 2012 interview Peter provided some insight into what gave him the greatest sense of accomplishment in his work:

In probability theory I think I am probably most pleased with my work on the details of rates of convergence in the central limit theorem, although it has received very little attention from anyone else! I am also pleased with my contributions to continuum percolation. I think my best work in statistics is about properties of bootstrap methods, explaining why some approaches are to be preferred to others.⁴

We will divide the immense corpus of work into several somewhat artificially distinct sections that we hope will give some idea of the extent of Peter’s influence on so many areas of statistics. We consider first his early work in probability theory, in classical limit theorems, in extreme value distributions and in stochastic geometry. We follow this with some comments on his work on the bootstrap that was much influenced by his early work on probability and was the initial source of the acclaim accorded his work. Next we consider his work in nonparametric function estimation that remained



Figure 5. Peter with his friend and collaborator Michael Titterington and Michael's son David, Scotland, 2005.

of interest to him for many years, then high dimensional data and functional data analysis and finally on deconvolution. The order of these sections is roughly the order in which he first considered problems in the area. He frequently returned after several years to an area of earlier interest.

Martingales and Rates of Convergence

Immediately following his undergraduate years, Peter produced novel results of importance in the theory of martingales and in the classical theory of rates of convergence to the central limit theorem. Peter had an immense admiration for the work of Russian probabilists, in particular, for Petrov's 1975 book.⁵ He said that this would be his choice if he were allowed to choose only one book to take with him to a desert island. His work, not only in these classical areas of limit theorems, but in his approach throughout his career, reflected this appreciation of the beauty to be found in rigour and style of mathematical presentation.

In his short period at the ANU in 1974 he commenced work on martingales under the supervision of Chris Heyde. The work was written as a master's thesis to be submitted two years later. He continued this work at Oxford with a section of his thesis on martingale invariance principles that was published as his first important work.⁶ Martingales had been extensively studied by several outstanding authors in the years immediately before Peter's work and his limit results here produced a functional central limit theorem under weaker conditions for sums of martingale differences normalized by the square root of the sum of their squares. More work

was produced in collaboration with Chris Heyde, culminating in their definitive monograph.⁷

For over a decade from commencing work for his thesis at Oxford, Peter obtained a large number of results on rates of convergence in the central limit theorem, published in more than 25 papers and his monograph.⁸ These were important both for their intrinsic interest in refining classical results on convergence by seeking minimal conditions and for the preparation their development afforded him for much of his later work, particularly his work on the bootstrap. The book on rates of convergence used a novel leading term approach, related to asymptotic expansions, to obtain improvements on the rates of convergence previously obtained. The main concern is with obtaining refinements to both absolute and non-uniform errors in bounds to the convergence rate for sums of independent random variables in triangular arrays, first using standardization by the expectation and variance, then on examining improvements in the rate by more general standardization. In considering both upper and lower bounds on the convergence rate based on truncated moments he generalized and improved the characterizations obtained in the two decades before this work.

This interest in rates of convergence resulted in important results when he returned to the classical area of rates of convergence for the central limit theorem by applying his leading term approach again. He showed that using a specific sequence of norming constants, other than the mean and standard deviation, could lead to optimal rates of convergence,⁹ and he considered random norming in several papers including in 1988,¹⁰ and in one of his later papers on the area.¹¹ He also used the leading term approach in

rates of convergence generalized to asymptotic expansions,¹² and to convergence in the multivariate central limit.¹³ Another set of brilliant results introduced by Peter contained his results on convergence determining sets.¹⁴ It is truly remarkable that Peter was able to obtain such novelty in his approach to an area that had been central to probability theory for several decades before his time.

This whole series of results represents just the beginning of Peter's use of asymptotic methods of probability that were used throughout his subsequent work in both probability and statistical theory.

Extremes

Peter began working on extremes, the largest and smallest observations in a sample, and near-extremes in his sixth and seventh papers.¹⁵ These and his other early papers on extremes are contributions to probability theory that fit well with his contemporaneous papers; the concern was with the study of limiting distributions and rates of convergence. This work provided specific results and a strong technical foundation for Peter's later statistical work on extremes that began with estimating parameters of regular variation,¹⁶ and the endpoint of a distribution.¹⁷

Peter studied the properties of Hill's estimator of the exponent of regular variation.¹⁸ The estimator is obtained as a conditional maximum likelihood estimator under a simple model. Peter's contribution was to study the properties of the estimator assuming that the adopted model holds only approximately and only in the tails of a distribution. This was later followed up by work on minimax rates of convergence and, since the estimator depends on the r largest order statistics, several papers on choosing r . One of these papers considered choosing r when we want to estimate probabilities and quantiles beyond the range of the data.¹⁹ They transformed the data to bring the quantities we want to estimate within the range of the transformed data, used the m out of n bootstrap to choose r and then used a regression method to extrapolate the estimates back to the original scale. The last paper on choosing r , considered a different approach in which r is increased until the bias has a significant effect on the asymptotic approximation to the sampling distribution of Hill's estimator.²⁰

Peter considered the problem of estimating the endpoint of a distribution.²¹ Peter used a similar approach to that which he used earlier: he chose a specific simple density for the r largest observations, maximized the conditional likelihood to construct an estimator of the endpoint and then studied the properties of the estimator when the model only holds approximately near the endpoint.²² Peter also worked on the higher dimensional version of endpoint estimation known as frontier or boundary estimation. He incorporated his other research interests into these problems using the iterated bootstrap to calibrate bootstrap confidence intervals for the frontier,²³ and allowing errors-in-variables to complicate estimation of the frontier.²⁴

Peter also worked on more conventional problems in extremes applying the local likelihood approach to estimate temporal trends when fitting parametric models to weakly dependent time series data.²⁵ For bivariate extremes, parametric estimates of the marginal distributions and nonparametric estimates of the dependence function were developed,²⁶ enabling the construction of joint and

conditional prediction regions for extreme events that use the bootstrap to calibrate the prediction regions and obtain the desired asymptotic coverage.²⁷

Coverage Processes

Two remarks, both related to work in this area, are worth including as an introduction to Peter's work on coverage processes, which it appears originated from conversations with Pat Moran and Roger Miles immediately following his arrival at ANU at the end of 1973. Steve Stigler provided the following vignette illustrating the extraordinarily full understanding and clarity of mind which pervaded Peter's work:

In the early 1980s I was serving on an editorial panel for the Wiley series on statistics, and with no advance warning I received a full book-length manuscript submission from Peter Hall, who at that time was not well known to me. I was astounded—the MS was entirely neatly handwritten and showed very few corrections or cross-outs. I was astonished by the quality of mind that could conceive such a work in its entirety and write it out as if at one sitting. I had no doubt it was the first and only draft, and worried that it was also the only copy. I sent the precious MS on to the editor, Bea Shube, and it was published as Peter's book on coverage processes.²⁸

Persi Diaconis gave the following comment that also sums up the essential originality of Peter's thought:

I have looked seriously at dozens of Peter Hall's papers over the years. One thing that struck me, there is always a twist or trick or special case singled out that is charming and leaves you saying 'How did he think of that?' As an example, take his work (book) about coverage processes. He solved a long-open problem: If you drop random small caps on a sphere, how long does it take to cover all (or most) of the sphere? He not only gave limit theorems, but supplemented this with useful versions of his formulas and counter-examples to show that his assumptions were necessary. The whole is presented in a user-friendly package, along with extensions that went well beyond the base questions (e.g. to manifolds and varieties of caps).²⁹

Our remarks on Peter's work in this area have been much influenced by extensive notes from Adrian Baddeley FAA. Peter's approach to stochastic geometry was founded on asymptotic approximations from probability theory applied when considering large numbers of sets, rather than on the traditional approaches using exact results or multi-dimensional calculus. Most of these results were obtained in the mid-1980s and many were collected in his book on coverage processes.³⁰ Peter maintained an interest in this area returning to it with new ideas for many years.

Questions of interest arise by considering overlapping random sets distributed randomly in k -dimensional space. In a first result in k -dimensions, Peter obtained the mean and variance of the vacancy, or content of uncovered regions, for randomly located interpenetrating spheres of unit radius.³¹ These were used to obtain limit theorems for vacancy.³²

In one of the most striking results, connected uncovered regions were considered in a high intensity setting.³³ A complete description of limiting distributions of the size, shape and number of the uncovered regions was given. These results were then used to obtain the limiting distribution of vacancy, together with the probability of total coverage.

The nature of clumps, connected regions formed by overlapping random sets distributed randomly in k -dimensional space, was

another subject of interest;³⁴ Peter considered infinite clumping, or percolation, and gave conditions for the existence of a critical intensity for formation of clumps with infinite mean size. Then limit theorems were obtained for clumps arising from both moderate intensity and sparse mosaics.³⁵

Bootstrap

Peter Hall's work on the bootstrap was the culmination of his early interest in classical limit theory, which occupied much of his attention for the first decade of his career. This early work permitted him to develop a remarkably complete asymptotic theory associated with the bootstrap. His work on rates of convergence to the central limit theorem used an approach related to the Edgeworth expansions and was well developed by the time he turned his attention to the bootstrap. The bootstrap has proved central to subsequent statistical applications.³⁶ Brad Efron, who originally proposed the bootstrap, commented on Peter's work as follows:

Perhaps I am guilty of bias in favoring Peter's bootstrap work, though it is only a small portion of his 600+ published papers. It is easy (for me) to forget how much hard work was involved in putting the bootstrap on a firm footing. Hall's 1988 *Annals of Statistics* paper marked a key moment.³⁷ It and its extensive discussion occupied 68 pages of the September issue. One can get a good feeling for the interest aroused from the comments of the 10 discussants—nothing pro forma here, everyone had salient points to make. That kind of interest is a sure sign of a paper's importance. Peter's verification of second-order correctness for bootstrap confidence intervals definitely raised my spirits—I had been hoping for just such results. All of this was brought successfully home in *The Bootstrap and Edgeworth Expansion*,³⁸ a book that maintains its honored place on my short shelf.³⁹

In essence, Efron's bootstrap⁴⁰ approximates statistics calculated from a sample from the true unknown distribution, by statistics calculated from samples from the empirical distribution of the observed sample. Tests of significance were to be considered, but the primary focus was on confidence intervals. An inversion of Edgeworth expansions,⁴¹ although not concerned with the bootstrap, seems to have been an ideal introduction to the methods used in Peter's celebrated works on the bootstrap.⁴² In these, comparing Edgeworth expansions and their inverses for smooth functions of means for both the true and bootstrap distributions, he showed that confidence intervals and tests based on Studentized means were second order correct. He showed that this property was shared by Efron's accelerated bias correction method, but not by other methods that had been proposed. There were several approaches to bootstrapping that had been proposed and this work gave definitive answers and clarity to the questions of which bootstrap proposals were superior. There were several papers on asymptotic approximations to the bootstrap that predate Peter's work, two of which should be mentioned.⁴³ However, his work gave such a comprehensive answer to the questions concerning bootstrap accuracy that they take the central place.

In the previous paragraph Peter's early work on the bootstrap was described. However, this was but the tip of the iceberg. We find 84 papers with bootstrap in the title and others essentially on aspects of the bootstrap. It is not possible here to mention the full extent of this work, but reference must be made to double bootstrap, iterated bootstrap and block bootstrap, which greatly widened the role played by the bootstrap and which appeared in Peter's work until 2015.

Applications, always together with rigorous theory, in nonparametric regression, density estimation, functional data analysis and multiple testing were instances of the incredible breadth of understanding that Peter brought to this subject. A more comprehensive discussion of Peter's role in the bootstrap was given by Chen.⁴⁴ One additional aspect of Peter's work on the bootstrap was his early treatment of the relative error for standardized means using the Cramér large deviation theory to show that the bootstrap outperformed Edgeworth approximations in most instances.⁴⁵

Nonparametric Function Estimation

Nonparametric function estimation was one of the first areas of statistics that Peter became interested in and he continued working in the field throughout his career. Peter was so expert and powerful in this field that, if he had not also worked on the bootstrap, it may well have been recognized as the field of his main contributions to statistics.

The problem of interest in nonparametric function estimation is to estimate a function such as a density function or a regression function or its derivatives when the function is specified, not in terms of a finite number of unknown parameters, but as belonging to a particular, general class of unknown smooth functions. The challenges in the problem arise from trying to estimate a general function (an unknown infinite dimensional parameter) from a finite dataset. The estimators are typically biased so an important starting point is to approximate measures such as the mean squared error or the mean integrated squared error to evaluate accuracy and precision. Peter was always interested in the rates of convergence of estimators and he readily demonstrated his power and expertise in building on the work of Farrel, Stone, Ibragimov and Has'minskii⁴⁶ to establish these for nonparametric function estimators in different problems.

Peter's first papers on density estimation used truncated series expansions,⁴⁷ but he quickly moved to kernel smoothing methods and later wavelets. These methods depend critically on the choice of a tuning parameter, which for kernel estimators is the bandwidth. Peter worked on cross-validation and direct estimation (or 'plug-in') approaches to estimating bandwidths. Two important works obtained remarkable results on cross-validation.⁴⁸

Modern nonparametric function estimation dates from the 1950s with important contributions by Rosenblatt, Whittle, Parzen, Watson and Nadaraya.⁴⁹ According to the interview Peter gave in 2015, he was attracted to do research on kernel density estimators by a paper by Eve Bofinger who he met when she and her husband Vic Bofinger visited ANU while Peter was a masters student there.⁵⁰ In 1975, Eve Bofinger published two papers on using order statistics to estimate the density evaluated at a quantile of the underlying distribution⁵¹ so it is not clear what paper Peter was referring to. Nonetheless, it is clear that she had an influence on Peter's later research directions.

Other challenging problems also attracted Peter's attention. Boundary effects occur when the bias of an estimator of a nonparametric function has a different order near a support boundary than it has in the interior of the support. A method of constructing pseudodata beyond the boundary that makes the order of the bias for standard estimators the same across the entire support was proposed.⁵² Estimation under shape constraints (such as monotonicity) adds additional challenges to nonparametric function estimation; a general data sharpening technique involving data

perturbation that performs well in this context was suggested.⁵³ For residual variance estimation, elegant results were obtained on the effect of estimating the mean function on estimating the residual variance.⁵⁴

Projection pursuit regression⁵⁵ and single index models⁵⁶ are methods of nonparametric regression that involve estimation of both parametric (direction vectors) and nonparametric (smooth functions) components. Peter applied kernel estimators in projection pursuit;⁵⁷ he used a two-stage estimation scheme with two different bandwidths in the two stages. Later, a simultaneous estimation procedure with a single bandwidth for single index models was proposed.⁵⁸

A more detailed review of the breadth and depth of Peter's work on nonparametric function estimation is given by Cheng and Fan.⁵⁹

High Dimensional Data

Questions involving observations in high dimensions with moderate sample size have become of central interest since the advent of computers has permitted such considerations. Two of Peter's papers have had a profound effect on this major field. Both of these are discussed by Samworth,⁶⁰ who gives more detail on them and on other high dimensional work but we will give some description of these results here.

Hall and Li considered a random vector in p dimensions with zero expectation and unit covariance matrix and showed that the regression of two random projections is nearly always close to linear when the dimension p is large enough.⁶¹ They related this to earlier work of Diaconis and Freedman showing that most low dimensional projections were normal, and noted that normal densities have linear conditional expectations.⁶² They also gave several applications of this extraordinary result and it has since had wide ranging implications.

A geometric representation of high dimensional data was given when the dimension becomes large and the sample size remains fixed.⁶³ It was shown, under some conditions which imply that a law of large numbers applies to the mean of the variances of elements of each random vector, that the scaled random vectors are asymptotically located on the vertices of a simplex centred at the expectation. Further, it was shown that for two samples of sizes m and n , for each of which the scaled vectors are asymptotically located on the vertices of a simplex, that each pair with one scaled vector from each sample are equidistant. This geometric representation allowed analysis of some discrimination methods such as the support vector machine and distance weighted discrimination and provided a theoretical explanation of some 'previously puzzling phenomena'.

Peter's work on classification techniques using distances between high dimensional vectors can appropriately be considered here. We comment on only two related papers,⁶⁴ in which scale adjustment for classification using distances in high dimensions was proposed for methods including the centroid and support vector machine and shown to produce asymptotically optimal classification.

Functional Data Analysis

Functional data comprises samples of random functions or stochastic processes. In practice, observations are recorded at only a finite number of points for each function, but the central feature is the study

of the functions themselves. Thus, the objects of interest are infinite dimensional and dimension reduction is an essential feature of their study. Functional data analysis, the analysis of functional data, was popularized in the monograph by Ramsay and Silverman.⁶⁵ The brief description here owes much to the detailed discussion of Peter's work in functional data analysis in Müller's review.⁶⁶ (See Fig. 6.)

In Peter's first paper in this area,⁶⁷ methods for estimating the density and mode for random curves were developed. The approach was based on finite dimensional approximations of generalized Fourier expansions using an empirical basis; this allowed the authors to work with the finite dimensional density of the approximating Fourier coefficients. Even then the density is difficult to deal with but the mode can be estimated using kernel methods. An algorithm for estimation was proposed and it was suggested that this could be used for clustering curves.

Principal component analysis is a major method for dimension reduction and such methods have been proposed for functional data. In 2006, two papers appeared applying the bootstrap to problems in functional principal components. One obtained asymptotic expansions for estimates of the eigenvalues and eigenfunctions for functional data and used this approach to quantify the accuracy of the estimates via bootstrap methods,⁶⁸ and the other investigated whether finite dimensional methods are useful approximations.⁶⁹

The first of several papers on functional regression considered prediction using an estimate of the slope function based on functional principal components and established rates of convergence for the predictors.⁷⁰ It was noted that under some smoothness conditions the rate of convergence for the estimate of the prediction is the same as for the finite dimensional problem and much better than the rate for the estimate of the slope function. These papers had a profound effect on subsequent developments in the area.

In a remarkable paper,⁷¹ near perfect discrimination (separation into groups) was shown to be possible for functional data, using partial least-squares or projection onto a finite number of principal components. This result, stemming from the high dimensional nature of functional data, is in contrast to inefficient discrimination achieved by analogous methods in classical multivariate analysis, where the asymptotic results depend on increasing sample size with the dimension held fixed.

Deconvolution

Nonparametric density estimation is complicated when, rather than directly observing random variables (data) from the density of interest, the observations are the sum (convolution) of a random variable from the density of interest and an independent (error) random variable from some other distribution. Similarly, nonparametric regression estimation is complicated when, rather than observing the explanatory variables directly, the observations are the sum of the explanatory variable and an independent (error) random variable from some other distribution. These kinds of errors-in-variables or deconvolution problems require us to separate the effects of the errors from those of the variable of interest. The solutions depend on what additional information (that is, what we know, can estimate or are willing to assume) about the error random variables can be brought to the problem; trying to minimize this information gives rise to the kinds of challenging technical nonparametric problems that interested Peter.



Figure 6. Peter, Hans Müller and Qiwei Yao, Workshop on Analysis of High-Dimensional and Functional Data in Honour of Peter Hall, UC Davis, 2012.

Peter's first work on deconvolution built on work of Stefanski and Carroll to establish minimax rates of convergence for nonparametric estimators of the density of interest.⁷² In particular, for normally distributed error random variables, showing that the minimax rate of convergence is only logarithmic. It was later found that this very slow rate can be improved by assuming that the error variance decreases to zero as the sample size increases.⁷³ In one of his last papers,⁷⁴ it was shown how to estimate the density of interest when it is sufficiently irregular (it cannot be expressed as a mixture of a symmetric density and another density) to be distinguished from the unknown, symmetric error density. The presence of the errors makes smoothing parameter estimation more difficult in deconvolution problems than in standard nonparametric function estimation problems. A general simulation-extrapolation method was proposed for estimating smoothing parameters that produces consistent nonparametric curve estimators.⁷⁵ In fact, in his research, Peter considered a range of different deconvolution problems under different conditions, as well as different approaches to solving them with detailed discussion given in Delaigle's review.⁷⁶

Epilogue

Peter's untimely death brought a torrent of expressions of grief from around the world. In addition to the universal plaudits accorded his brilliance, he was widely admired for his generosity of spirit, the kindness he displayed to colleagues and students and his gentle nature and modesty. In a note to Jeannie, Mark Westcott, perhaps Peter's closest friend, wrote: 'Despite Peter's towering achievements, formidable intellect and workload, his most remarkable legacy is the

gentleness, care and kindness within which his professional life was lived'.⁷⁷

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