Report of Subcommittee I

Steering Committee for the development of

The Mathematical Sciences in Australia
A Vision for 2025

The Decadal Plan for the Mathematical Sciences 2016-25

Mathematics and statistics education
in schools and colleges

Subcommittee 1: Mathematics and Statistics Education in Schools and Colleges:

Draft Recommendations – with rationale

The overarching goal is to achieve:

World leading mathematics teaching and learning for all Australian students

Key to achieving this goal is to increase the supply of well-qualified teachers of mathematics, ensuring that all secondary school students have access to specialist mathematics teachers and improving the overall achievement of Australian students within an equitable mathematics education system.

The recommendations that follow are directed towards this goal.

Recommendation 1:

Improve teacher education for mathematics

Initial teacher education for teachers of mathematics should use evidence based practices to develop pre-service teachers' relevant knowledge and affective characteristics. It should be characterised by collaboration among mathematicians, mathematics education researchers, and the mathematics teaching profession. Mathematics teacher educators should have appropriate qualifications and school teaching experience.

Australian Initial teacher education (ITE) programs for teachers of mathematics at all levels of schooling should be:

a. focused on the development of pre-service teachers' mathematical content knowledge, pedagogical content knowledge for teaching mathematics and their beliefs about and attitudes to the discipline,
b. based on evidence of effective practices in initial teacher education for teachers of mathematics,
c. comprise mathematics content units taught by mathematics teacher educators (within or outside of education entities) with and initial degree in mathematics and experience of teaching at the level for which they are preparing pre-service teachers to teach, and
d. characterised by collaboration among mathematicians, mathematics education researchers, and the mathematics teaching profession.

ITE includes that conducted in Faculties or Schools of Education in higher education institutions leading to teaching qualifications recognised by the Australian Institute for Teaching and School Leadership (AITSL). For secondary mathematics teachers it often includes undergraduate mathematics study in organisational units outside of Faculties or Schools of Education. ITE thus encompasses all tertiary study leading to recognised qualifications to teach in early childhood contexts, and primary or secondary schools. Ensuring that Australian initial teacher education is first class will require attention to

curriculum and structure of ITE programs, the characteristics of prospective teachers entering ITE programs, and the characteristics of teacher educators both within and beyond Education entities.

These variables, considered as inter-related and each influencing the outcomes of ITE, framed the Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto, Schwille, Senk, Invargson, Peck, & Rowley, 2008). An adaption of the TEDS-M framework, shown in Figure 1, was used as the conceptual model for an Australian study of ITE for teachers of mathematics (Beswick & Callingham, 2011). Because TEDS-M was an international study, Tatto et al. (2008) included in their model national context in terms of relevant policies, schooling and social factors. Such factors must be important considerations in discerning what can be learned from models of ITE in other countries.

High quality mathematics teaching requires comprehensive development of knowledge for teaching mathematics at the ITE level. An important inclusion is Shulman's (1987) knowledge types in the context of mathematics teaching: knowledge of mathematics; general pedagogy; pedagogical content knowledge for teaching mathematics; knowledge of how students learn mathematics; knowledge of curriculum, knowledge of educational contexts; and knowledge of ends and purposes that underpin the teaching of mathematics. In addition, teachers need to also develop belief sets about mathematics teaching that are compatible with provision of high quality mathematics learning for all students. Fundamental is the belief that all students can learn mathematics. Appropriate access for all students to the entire mathematics curriculum is imperative. ITE should also engender in teachers an appreciation of the discipline of mathematics and belief in its inherent value as an object of study and realm of creative endeavour, and its value to society.

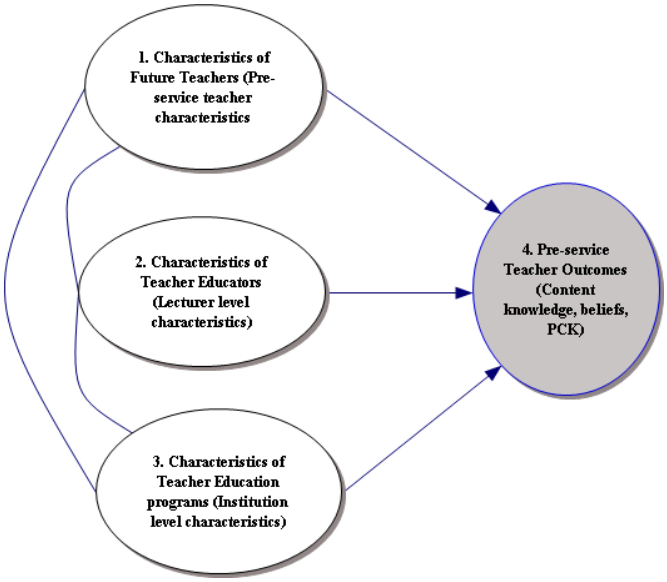

Figure 1. Conceptual framework for improving the effectiveness of teacher preparation for mathematics teaching (adapted from Tatto et al., 2008, p. 15)

A comparative study of final year pre-service secondary teachers involving six countries, Mathematics Teaching in the 21st Century (MT21), found considerable variation in ITE both within and between countries in terms of the opportunity to learn (measured by the amount

of time) mathematics content, mathematics pedagogy, and general pedagogy, as well as time spent in university study and school based experience provided by ITE (Schmidt et al., 2008). Beswick and Goos (2012) similarly reported a variety of program structures for Australian prospective primary teachers. Interestingly, their measures of mathematics content knowledge and mathematics pedagogical content knowledge did not show any differences according to the structure of delivery mode of their ITE program. Although the need for more nuanced measures was acknowledge (Beswick & Goos, 2012) a reasonable inference from the relative uniformity of outputs is that the variety of ITE programs offer reflect the variation among entering cohorts.

If Australian ITE programs are to produce world-leading teachers then attention is needed to improving the quality of all ITE without prescribing a single model. Conceptualisations of teacher competence such as that used in the TEDS-M study and described by (Döhrmann, Kaiser, & Blömeke, 2012) are helpful in drawing attention to the domains to be considered.

Efforts to improve ITE must be collaborative, including the range of stakeholders involved in their delivery (mathematicians and mathematics educators) and concerned with their outcomes (employers and professional associations). The importance of collaboration between mathematicians and mathematics educators emphasised by Seaman and Szydlik (2007) who claimed that teachers of pre-service primary teachers in particular must themselves be enculturated in both the community of mathematics teachers and the community of mathematicians. That is they must have both an appreciation of mathematical ways of thinking and knowing and also of the students' intuitive understandings, the school curriculum and appropriate tasks, representations and examples for teaching primary school mathematics. This would necessarily also be the case for ITE of pre-service secondary mathematics teachers. Further, there is evidence that pre-service primary teachers make better progress in their mathematics content knowledge when they undertake mathematics content units designed specifically for them rather than generic mathematics content units (Matthews & Seaman, 2007), and where the teacher actively engages students in learning the content (McCrory, Zhang, Francis & Young, 2009) modelling the kinds of pedagogies that the pre-service teachers would be expected to use (Hodgson, 2001). Again, the case would be the same for pre-service secondary mathematics teachers. Interestingly, the preparation of pre-service teachers in Singapore focuses on the school mathematics topics that teachers will be teaching rather than higher levels of tertiary mathematics as it is well-regarded that more tertiary mathematics does not necessarily translate into better understanding of teaching (Yoong, 2009). This fact needs to be balanced with the need also for teachers at any grade level to have the knowledge that Ball, Thames and Phelps (2008) have termed Horizon Content Knowledge – that is, knowledge of how mathematical ideas are connected and developed across the school years including those beyond the year levels at which a teacher is qualified to teach. Ideally instructors of pre-service teachers would have both at least one degree in mathematics and experience in teaching primary school mathematics (Seaman & Szydlik, 2007), and similarly pre-service secondary mathematics teacher education should consider the amount of time, focus and staff associated with ITE for that cohort.

Based on 825 responses to a survey of US higher education institutions offering ITE, Masingila, Olanoff and Kwaka (2012) identified a lack of professional learning opportunities for university

teachers of mathematics content for pre-service primary teachers despite widespread interest in such opportunities. They pointed out that isolation in separate departments militated against opportunities to talk and collaborate and hence many were unsure of how to implement calls to actively engage the pre-service teachers in their mathematics learning. Masingila et al. (2009) recommended the establishment of communities of practice to facilitate collaboration with a view to improved ITE in relation to mathematics content.

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Recommendation 2:

Increase and sustain Australia's commitment to ongoing, systematic, high quality, mathematics specific professional learning for teachers of mathematics at all levels of schooling

Professional learning should be based on identified needs and robust research. All primary and early childhood teachers should have ready access to relevant expertise in mathematical content and mathematics pedagogical content knowledge, and out-of-area secondary teachers must be provided appropriate retraining relevant to their existing qualifications. All secondary students should be taught mathematics by appropriately qualified specialist mathematics teachers.

The necessary commitment to professional learning should:

a. be based on a national audit of professional learning needs of teachers of mathematics and existing programs, and including the identification of areas where further research is required in order to provide robust and practical advice to teachers
b. ensure that all primary and early childhood teachers have ready access to expertise in mathematical content and mathematics pedagogical content knowledge relevant to their contexts
c. provide opportunities for out-of-area teachers of secondary mathematics to engage in formal re-training in courses appropriate to their existing qualifications
d. be directed towards ensuring that all secondary students are taught mathematics by appropriately qualified specialist mathematics teachers

Ongoing professional learning that is focussed on improving teaching and learning is a hallmark of high performing education systems. It is a requirement for all teachers throughout their careers to stay abreast of developments in research concerning effective mathematics teaching, curriculum developments, and to share effective practice and engage in collaborative planning and other activity directed at improved teaching and learning. The particular needs of beginning teachers and out-of-area teachers must be attended to with additional support to

become integrated into the profession of mathematics teaching and to develop repertoires of knowledge and practice necessary for effective teaching of mathematics. Meeting workforce demands for appropriately qualified teachers of mathematics means that for the foreseeable future professional learning for out-of-area teachers will need to include formal re-training in courses appropriate to the existing qualifications of these teachers.

Primary and early childhood teachers are necessarily generalists. These teachers must all have ready access to expertise in mathematical content and mathematics pedagogical content knowledge relevant to their contexts. Secondary teachers with an undergraduate mathematics degree also require opportunities to maintain the currency of their knowledge and understanding of mathematics, which is a dynamic discipline.

Anecdotally (including in submissions to this subcommittee), there is evidence of needs for professional learning in relation to specific topics. Examples include: the teaching of statistics in meaningful ways that provide students with the capacity to be quantitatively literate citizens whilst equipping them with the conceptual foundations for further study of statistics; and effective teaching of mathematics classes catering for students with diverse histories of mathematical attainment, particularly in secondary school. We recommend a systematic approach to the provision of professional learning based on a national audit of needs and existing programs, and including the identification of areas where further research is required in order to provide robust and practical advice to teachers.

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Recommendation 3:

Establish career paths for teachers of mathematics that will enhance the status of mathematics teaching, develop a pool of expert teachers of mathematics, keep the best teachers of mathematics engaged in the teaching of mathematics, and contribute to improving the attractiveness of teaching is a career.

Career paths for teachers of mathematics should:

a. provide excellent teachers of mathematics with opportunities for promotion that allow them to lead mathematics teaching and learning at school level, across school clusters, and at regional and state/territory levels, and
b. include a mathematics leader in every school.

Enhancing the status of mathematics teaching will address issues of both attracting and retaining well-qualified teachers of mathematics. Shortages of mathematics teachers are well recognised and increasingly acute (Australian Mathematical Sciences Institute, 2013) particularly in regional and remote schools (Lyons, Cooksey, Panizzon, Parnell, & Pegg, 2006).

They result from both the undersupply of specialist secondary mathematics teachers and high rates of attrition in the early years of teaching, particularly from hard to staff schools in which graduate teachers are over-represented. Research in the US has shown that rates of attrition are affected primarily by working conditions (e.g., availability of resources and administrative support, teaching loads, poor quality of facilities, and perceived lack of influence on school organisation) and salary differentials, with student demographics making a much smaller contribution to teachers' decisions to leave the profession (Darling-Hammond & Sykes, 2003). Darling-Hammond and Sykes (2003) noted that the difference in salary for graduates who become teachers compared to those with the same qualifications who pursue other careers is greater for mathematics and science teachers than for teachers of other subjects such as English and social studies. Well-qualified mathematics teachers have marketable skills and hence options that make the provision of satisfying working conditions and rewarding career paths imperative.

Excellent teachers of mathematics need opportunities for promotion that allow them to lead mathematics teaching and learning at school level, across school clusters, and at regional and state/territory levels. This will require a commitment to: (1) enhance the status of mathematics teaching by rewarding the development and sharing of expertise, (2) develop a pool of expert teachers of mathematics able to contribute to the professional development of colleagues, (3) keep the best teachers of mathematics engaged in the teaching of mathematics, and (4) contribute to improving the attractiveness of teaching is a career for those with appropriate mathematics qualifications. Further, every school (early childhood, primary, and secondary) must have a mathematics leader whose responsibility includes leading ongoing improvement of mathematics teaching and learning in that school.

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Recommendation 4:

Raise expectations for the mathematics achievement of all Australian students

Australian students are capable of mathematics achievement that is the equal of students anywhere in the world. Our students should be able to succeed in mathematics wherever they live, and whatever their ethnicity or their gender. Achieving this goal involves eliminating

practices that convey low expectations of mathematics achievement to students, and enabling students to develop a belief in their capacity to solve mathematics problems and to learn mathematics.

The high expectations of required will be achieved by:

a. eliminating practices that convey low expectations of mathematics achievement to students, and
b. equipping teachers at all levels of schooling to use pedagogies that provide appropriate challenge for all students, thereby allowing students to experience productive struggle with mathematics, build resilience and develop a belief in their capacity to solve mathematics problems and to learn mathematics.

High teacher academic expectations are associated with higher student achievement (e.g., Archambault, Janoz, & Chouinard, 2012) but there is evidence to suggest that teachers in disadvantaged schools have lower expectations for the academic achievement of their students. For example, Thomson and Hillman (2014), reported that PISA 2012 data show that schools with more than 25% of their students from disadvantaged backgrounds and fewer than 25% of students from affluent backgrounds place less emphasis on academic success than schools that are not disadvantaged. This and related factors are associated with lower student mathematics achievement amounting to 58 points in PISA 2012 (Thomson & Hillman, 2014). An important part of setting students up for success in mathematics in subsequent years is to develop their self-concept and self-efficacy in relation to mathematics learning. Both of these variables were strongly related to achievement in mathematical literacy in PISA 2012 in Australia and across the OECD (Thomson, de Bortoli, & Buckley, 2013). Appropriate challenge that allows students to experience ultimately productive struggle, along with judicious use of praise, helps to build resilience and the belief in one's capacity to solve mathematics problems and to learn mathematics (Dweck, 2002). Teachers are unlikely to allow students to struggle with mathematics or to provide supports that maintain the cognitive demand of a task (such as in ways described by Henningsen and Stein, 1997) if they do not believe the student will be able to solve the problem.

High academic expectations are important for all students, including those who struggle to learn mathematics and those who show exceptional aptitude for the subject. The objective must be to ensure that all students regardless of social background, gender, location, prior experience or attainment are taught in such a way that they are able to realise their mathematical potential.

One way in which expectations of students' attainment in mathematics are reflected plainly is in decisions that place students in differing classes for mathematics according to their perceived ability or prior attainment. Such practices are referred to as streaming. Streaming for mathematics learning is common from Year 9 in Australian schools but often occurs earlier. Streaming at any level of schooling is often justified in terms of being able to provide teaching that better meets the differing needs of particular students – consolidating the understanding of low attainers and extending high attainers. Although there is evidence that Australian mathematics teachers support streaming particularly from Year 9 (Anderson, 2010), research has shown that high quality learning environments can be created without streaming

(Boaler, 2006). In fact well taught heterogeneous groupings have advantages for all students in terms of both attainment and the development of positive attitudes (Boaler, 2006). Conversely streaming is associated with both cognitive and affective harms (Cahan & Linchevski, 1996; Linchevski & Kutsche, 1998; Zevenberger, 2005). A contributor to these impacts of streaming is likely to be teachers' expectations of students informed by their prior attainment, which is often from their placement in streamed mathematics classes and manifested in lower level tasks and poorer pedagogies even when the teacher is well-qualified and experienced (Beswick, 2004).

Streaming is typically described as ability grouping in primary schools and is typically a more flexible practice than that in secondary schools. Nevertheless, there is evidence that these practices can also be damaging, detracting from students self-efficacy as mathematics learners (Macintyre & Ireson, 2002). Despite the overall negative impacts of streaming short term interventions with the explicit aim of helping students to be able to participate fully in a mixed ability group can be effective. Examples of such programs include that reported by Gervasoni (2001) and Fuchs, Fuchs, Powell, Seethaler, Crino and Fletcher (2008).

Gender differences in mathematics achievement have received considerable attention with the focus of research moving from helping girls to think differently, to the impacts of differential impacts of curriculum and other factors on boys and girls, to how teachers can create mathematics classrooms in which girls and boys can all succeed (Forgasz, Leder, & Vale, 2000). A necessary condition creating such classrooms is that the teacher has equally high expectations of boys and girls in relation to mathematics achievement.

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Recommendation 5:

Increase the sophistication of public discourse regarding mathematics and the role of teachers of mathematics.

Teaching mathematics well at any level of schooling is complex and demanding work that requires considerable knowledge and skill. The profession should continue and intensify its efforts to communicate what teachers of mathematics know and can do, using language that is accessible to lay people and that respects the informed perspectives of all stakeholders.

The public discourse can be influenced by:

a. more frequent and regular communication from the mathematicians, mathematics education researchers, and the mathematics teaching profession
b. communication that builds awareness of what teachers of mathematics know and can do, and
c. using language that is accessible to lay people and that respects the informed perspectives of all stakeholders.

School education is uniquely susceptible to uninformed commentary from anyone who has been a student in a school or a parent of a school student. The fact that mathematics is recognised as an important subject is appropriate but can add to the intensity of commentary in relation to its teaching. Teaching mathematics well at any level of schooling requires a great deal of complex knowledge and skill. The fact that expert teachers hold much of this tacitly has detracted from the ability of the profession to explain the nature of mathematics teaching. Recent and ongoing research into the knowledge required to teach mathematics is

contributing to making the complexity of expert mathematics teachers clearer and providing a vocabulary to explain it. It is important that the profession continues and intensifies its efforts to communicate what teachers of mathematics know and can do using language that is accessible to lay people and that respects the informed perspectives of all stakeholders. A commitment by government and bureaucracies to engaging in such conversations will allow them to make public contributions that enhance the status of mathematics teaching, improve public understanding of what it entails, contribute constructively to its enhancement, and make the profession of mathematics teaching increasingly attractive.

We see this recommendation as fitting within a broader goal of enhancing public understanding and discourse about the nature, role and contributions (social, economic, and cultural) of mathematics more broadly.