Report of Subcommittee II

Steering Committee for the development of

The Mathematical Sciences in Australia
A Vision for 2025

The Decadal Plan for the Mathematical Sciences 2016-25

Mathematics and statistics education
and training in universities

Decadal Plan for the Mathematical Sciences

Final subcommittee reports, March 2014

Subcommittee (ii)
Mathematics and statistics education and training in universities

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Introduction

Subcommittee (ii) was commissioned to report on undergraduate and coursework postgraduate mathematical sciences education. TAFE education was explicitly excluded from the brief, and assigned to Subcommittee (i), whose main business was school education. Research higher degree programs (master by research and doctoral programs were the responsibility of Subcommittee (iii), which addressed all aspects of mathematical sciences research in universities.

Subcommittee (ii) gave most of its attention to undergraduate issues. For pragmatic reasons, mathematical sciences majors attracted particular attention:

- the existence of honours or coursework postgraduate programs (and consequently the viability of research higher degrees) is critically dependent on an adequate supply of well-prepared, enthusiastic graduates with mathematical sciences majors;
- students who have proceeded at least as far as an undergraduate major are the graduates who will be attributed to the mathematical sciences profession, and their fitness for work or further study is a good measure of the health of university mathematical science education;
- mathematical sciences majors recruited into postgraduate teaching degrees are a promising source of future senior secondary mathematics teachers.

However, it would have been remiss of us to pay too little attention to the teaching of mathematical sciences material to broader cohorts ("service teaching"):

- modern society requires many university graduates who are not mathematical sciences specialists to be mathematically literate in a variety of ways, extending beyond basic numeracy, to any or all of software-empowered computational ability, appreciation of statistical concepts, being able to make sense of large data sets, being able to understand and critique mathematical models, and having specific beyond-school mathematical skills relevant to their profession;
- many future school teachers of mathematics (especially in the earlier secondary years) will have taken some university mathematics studies, but fall short of a major;
- mathematical science departments cannot survive only on the teaching income sourced from their own majors, and significant participation in service teaching is the only realistic alternative to starvation.

The subcommittee's findings and recommendations are discussed under the four themes that the subcommittee negotiated with the Steering Committee for the Decadal Plan.

Table 1: Classification of year 12 high-school mathematics subjects (terminology of Barrington and Brown [5] and Table 1.43 of Forgasz [9], updated for Western Australian changes).
Students taking an advanced subject are usually required to take the intermediate subject as well. NSW students taking Mathematics Extension 2 also must take the Mathematics Extension 1. Note that the coverage of subjects in the same column varies. The combination Mathematics plus Mathematics Extension 1 corresponds to what was formerly called three unit mathematics and falls a little short of most of the other advanced subjects, while taking all three of Mathematics, Mathematics Extension 1 and Mathematics Extension 2 corresponds to the former four-unit mathematics, which goes a little further than the other advanced subjects. For the ACT subjects, the "minor" versions fall short of most other subjects in the same column.

Authority	Intermediate	Advanced
ACARA future national curriculum	Mathematical Methods	Specialist Mathematics
ACT	Mathematical Methods (major/minor)	Specialist Mathematics (major/minor)
New South Wales	Mathematics [was 2-unit]	Math. Extension 1 [was 3-unit] Math. Extension 2 [was 4-unit]
Queensland	Mathematics B	Mathematics C
South Australia & Northern Territory	Mathematical Studies	Specialist Mathematics
Tasmania	Mathematics Methods	Mathematics Specialised
Victoria	Mathematical Methods (CAS)	Specialist Mathematics
Western Australia	Mathematics (3C/3D)	Specialist Mathematics (3A/3B) Specialist Mathematics (3C/3D)

Theme 1. Curriculum for specialist mathematics & statistics coursework-based programs

Some observations on the six questions identified as pertinent for Theme 1 are given below, but first we offer the following summary of the state of play for undergraduate majors. An undergraduate major in at least one area of specialization in the mathematical sciences is available at a majority of Australian universities, though the assessment of the exact situation is sensitive to the precise definition of what a major means. A minimalist definition would be based on 8 subjects in a 24-subject three-year degree, assuming at least "intermediate" final-year high school mathematics (see Table 1) and containing at least three subjects at third-year level. An example of a minimalist major is that offered by the University of the Sunshine Coast.[1] In their major, seven subjects are prescribed and choice is only possible between two third-year subjects. Outside of AMSI members, only five of ten institutions studied sustain at least a minimalist major (see Table 2), while 28 of 29 AMSI members offer at least minimalist majors.[2]

Where majors are not offered, the continuing existence of the mathematical sciences on the campus is in jeopardy and pathways to further studies in which mathematics and statistics pay a major role are closed. Minimalist majors give access only to a very restricted set of options for advanced study.

> **FINDING 1** The subcommittee finds the availability of undergraduate majors in the mathematical sciences is inadequate in regional universities and vulnerable or excessively narrow in scope in many capital city universities.

What might be done in practical terms to remedy this situation is addressed under Theme 3 later in this report.

[1] http://www.usc.edu.au/study/courses-and-programs/courses/faculty-of-science-health-education-and-engineering-courses (February 2014). This major appears to have grown from subjects created for Education degrees.

[2] The missing AMSI institution is the University of Canberra, which has a six-subject definition of a major, but does offer in total eight subjects in applied statistics, and a modest selection of subjects in mathematics per se. Swinburne University has significantly improved its previously restricted offering for students commencing in 2014.

Table 2: Mathematical Sciences majors offered in 2013 by universities that are not members of AMSI. Blank entries mean that a major was not offered in 2006 or in 2013.

(Data for ACU and Notre Dame from their web sites in February 2014; all other data provided by Jan Thomas. ACU and Notre Dame majors not available at some of their campuses.)

Institution	Some choice of flavour	Combined maths & stats	Applied maths
Charles Darwin University			
Murdoch University		yes	
University of Notre Dame	yes (in B.A.)		
Victoria University			lost since 2006
Southern Cross University			
Edith Cowan University		yes	
Central Queensland University		lost since 2006	
Bond University			
University of the Sunshine Coast			yes
Australian Catholic University	yes (in B.A.)		

1.1 *Is the match-up with school-leaver background appropriate (including streamed entry, vs one-size-fits-all, prerequisites vs assumed knowledge)?*

This question (and especially the matter of prerequisites) came to be seen by the subcommittee as of extreme importance, and our observations have considerable relevance also to matters considered by Subcommittee (i) and serious implications for our own Themes 1 and 4.

While some universities retain strictly enforced mathematics prerequisites for entry into some of their degrees, many have adopted a declared "assumed knowledge" (but no enforced prerequisite) approach for degree entry, but may or may not control entry to specific individual subjects post-enrolment, and some do not even declare assumed knowledge. To illustrate this, data concerning prerequisites for entry into several undergraduate courses in the eastern states and the ACT is provided in Table 3 (Victoria[3]), Table 4 (Queensland and northern NSW[4]) and Table 5, (New South Wales and ACT[5]). The Bachelor of Commerce (or the nearest equivalent), Bachelor of Engineering (choosing the Civil specialisation where entry is separated by stream) and Bachelor of Science were examined. Niche streams focussing on mathematics (actuarial programs in Commerce, and advanced mathematics programs in Science) were not considered. Programs based on initial enrolment via TAFE were not considered. Note that most institutions have some form of English language prerequisite, and this is excluded from the discussion. That is, a degree with no prerequisite except English is classified as having no prerequisites.

Victoria—Prerequisites survive here to a large extent in Engineering and Science. All Bachelor of Engineering programs have a mathematics prerequisite, all at the intermediate level [Mathematical Methods (CAS)].[6] For Science, intermediate mathematics is needed at Melbourne, Monash, Latrobe and RMIT; Victoria University and Swinburne will accept elementary mathematics on its own (Further Mathematics); Federation University asks for one subject only out of a list of seven that includes Mathematical Methods and Specialist Mathematics; and Deakin asks for nothing. Only Melbourne and Monash require intermediate mathematics for Commerce; Latrobe will accept elementary mathematics; and nobody else asks for anything.

[3] http://www.vtac.edu.au/CourseSearch/searchguide.htm

[4] http://www.qtac.edu.au/Courses/CourseHome.html

[5] http://www.uac.edu.au/undergraduate/course-search

[6] For many years, the University of Melbourne had been the last Victorian university requiring advanced mathematics (Specialist Mathematics) for undergraduate Engineering, but this requirement was dropped some years before Melbourne phased out undergraduate Engineering completely. The requirement was dropped in an attempt to boost female enrolments in the Bachelor of Engineering, but this was not particularly successful at the time.

Table 3: Sample undergraduate entry prerequisites for Victorian universities. For brevity "$SS \geq$" denotes "a VCE study score of at least". The mean study score in each subject is 30, with standard deviation ≈ 7. About 78% of all students completing a VCE subject have $SS \geq 25$, and about 93% have $SS \geq 20$. Also for brevity "MM" denotes "Mathematical Methods (CAS) Units 3 & 4" and "SM" denotes "Specialist Mathematics Units 3 & 4". Where "any maths" appears, the elementary subject Further Mathematics is accepted.

University	B Com (or equivalent)	B Eng (the Civil stream if disaggregated)	BSc
Australian Catholic	none [Brisbane and North Sydney]	not offered	any maths or any one of 7 other subjects [Exercise & Health Sci., Melbourne]
Deakin	none	$SS \geq 20$ in one of MM, SM	none
Federation [*former* Monash (Gippsland) & Ballarat]	none at year 12; year 11 maths (any) required	$SS \geq 15$ in one of MM, SM	$SS \geq 20$ in one of MM, SM, Biology, Chemistry, Geography, Physics or Psychology
La Trobe	$SS \geq 20$ in any maths [B Accounting/ Economics]	$SS \geq 20$ in Mathematical Methods (CAS) and $SS \geq 20$ in one of Specialist Mathematics or Physics	$SS \geq 20$ in one of MM, SM
Melbourne	$SS \geq 25$ in one of MM, SM	not offered at undergraduate level (entry via BSc)	$SS \geq 25$ in MM and $SS \geq 25$ in one of SM, Biology, Chemistry, Physics
Monash	$SS \geq 25$ in one of MM, SM	$SS \geq 25$ in MM and $SS \geq 25$ in one of Chemistry or Physics	$SS \geq 25$ in one of MM, SM, Biology, Chemistry, Geography, Physics or Psychology
RMIT	none [B Business]	$SS \geq 20$ in MM	$SS \geq 20$ in one of MM or SM [B Applied Science]
Swinburne	none [B Business]	$SS \geq 20$ in MM	$SS \geq 20$ in any maths
Victoria	none [B Business]	$SS \geq 20$ in MM or SM	$SS \geq 20$ in any maths

Queensland—For Commerce, only the University of Queensland lists a mathematics prerequisite; for some other institutions, mathematics is recommended or assumed knowledge, but the level assumed may be as low as the elementary subject Mathematics A. For Engineering, Queensland institutions usually prescribe the intermediate subject Mathematics B; school science subjects are also mandated in some cases. Prerequisites for the Bachelor of Science are more variable: the University of Queensland requires Mathematics B plus one of Chemistry or Physics; James Cook requires Mathematics B and (depending on the stream to be chosen) either Chemistry or one of Physics or Mathematics C; the University of Southern Queensland enforces Mathematics B for its Mathematics and Statistics stream. For all other Science programs it appears that there is either only assumed knowledge or recommended background, or nothing at all.

New South Wales—"Assumed knowledge" rules in NSW. None of the 10 Commerce programs or near equivalents[7] found in NSW had prerequisites: all either listed "assumed knowledge" or "recommended background". At least in documentation from the NSW UAC, the situation seems similar for Engineering and Science, with only Macquarie University listed as having some subject prerequisites, but not degree prerequisites on top of assumed knowledge on degree entry, and all other universities' programs shown as having "assumed knowledge". Macquarie's way of handling subject prerequisites not met from school is via bridging courses. For example, for the BE its web site[8] notes that "HSC

[7] Where a Bachelor of Commerce was not found, the following programs were considered instead: Charles Sturt—Bachelor of Business Management; UTS—Bachelor of Business; UWS—Bachelor of Business and Commerce.

[8] http://courses.mq.edu.au/undergraduate/degree/bachelor-of-engineering (accessed November 2013).

Table 4: Sample undergraduate entry prerequisites for Queensland and northern NSW. Note that "Mathematics A" is the elementary year 12 subject in Queensland. The descriptor "4,SA" refers to "sound achievement", the middle classification out of five criterion-referenced assessment levels in Queensland's moderated but individual school based result reporting.

University	B Com (or equivalent)	B Eng (the Civil stream if disaggregated)	BSc
Central Queensland	none: Maths A, B or C recommended	Maths B (4,SA); Physics recommended	none: Biology; Chemistry; Maths B recommended
Griffith	none	Maths B (4,SA); at least one of Physics, Chemistry or Maths C recommended	Maths A (4,SA) [Maths B some streams]; also at least one of Biology, Chemistry or Physics recommended
James Cook	none; Maths B recommended [B Business (Economics)]	Maths B (4,SA); at least one of Physics, Chemistry or Maths C recommended	Maths B (4,SA) and depending on major either Chemistry (4,SA) or one of Physics or Maths C (4,SA)
New England (Armidale)	none: Maths A, B or C recommended	Assumed knowledge: Maths B; Chemistry and/or Physics recommended	assumed knowledge: Maths B; depending on major - Biology, Chemistry and/or Physics recommended
Queensland	Maths B (4,SA)	Maths B (4,SA); one of Chemistry or Physics (4,SA)	Maths B (4,SA); one of Chemistry or Physics (4,SA)
QUT	not offered	Maths B (4,SA); at least one of Physics, Chemistry or Maths C recommended	assumed knowledge: Maths B (4,SA); least one of Biology, Chemistry, Physics, Earth Science or Maths C recommended
Southern Cross (Lismore)	none: Maths A, B or C recommended	none; Maths B and a science subject recommended	none: Biology and/or Chemistry; Geography; Maths B recommended
Southern Queensland	Assumed knowledge: Maths A (4,SA) [B Business]	Maths B (4,SA); Physics recommended	for Maths & Stats stream Maths B (4,SA); for Physical Sciences assumed knowledge Maths A (4,SA) with one of Biology, Chemistry or Physics recommended
Sunshine Coast	none: Maths A, B or C recommended [B Business]	Maths B (4,SA); Physics recommended	none: Maths A, B or C and a science subject (preferably Chemistry) recommended

Mathematics (Band 4) or its equivalent is a subject prerequisite for first year Engineering, Mathematics and Physics units (bridging courses are available)". UNSW Engineering lists[9] Mathematics Extension 1 and Physics as assumed knowledge for entry to the degree and individual subject handbook entries for UNSW first-year subjects give the sense that a given level of school achievement is assumed, but not policed. It is known that at the University of Newcastle, for a number of years stated prerequisites on subjects within degrees have not been policed.

Australian Capital Territory—Setting aside the UNSW Canberra campus at ADFA (where the UNSW arrangements apply), we find that the University of Canberra lists neither prerequisites nor assumed knowledge for its Bachelor of Commerce, Bachelor of Engineering in Network and Software Engineering and Bachelor of Science. ANU is more overtly demanding, listing assumed knowledge for Commerce, Engineering and Science, and also imposing actual prerequisites on some subjects, with bridging courses available for those who lack them.

[9] https://www.unsw.edu.au/sites/default/files/documents/2014cuttofflist_ATAR_Alevel_IB_OP.pdf.

Table 5: Sample undergraduate entry prerequisites (data from NSW UAC): NSW universities already listed in Table 4 are not included here.

University	B Com (or equivalent)	B Eng (the Civil stream if disaggregated)	BSc
ANU	assumed knowledge	assumed knowledge + prerequisites	subject prerequisites
Canberra	none	none [B Eng in Network and Software Engineering]	none
Charles Sturt	assumed knowledge [B Business Management]	not offered	assumed knowledge
Macquarie	assumed knowledge	some subject prerequisites	some subject prerequisites
New South Wales	assumed knowledge	assumed knowledge	assumed knowledge
Newcastle	assumed knowledge	assumed knowledge	assumed knowledge
Sydney	assumed knowledge	assumed knowledge	assumed knowledge
Univ. of Technology Sydney	assumed knowledge [B Business]	assumed knowledge	assumed knowledge
University of Western Sydney	assumed knowledge [B Business & Commerce]	assumed knowledge	assumed knowledge
Wollongong	assumed knowledge	assumed knowledge	assumed knowledge

Anecdotal evidence suggests that University administrations tend to prefer minimizing prerequisites for entry to degrees as a whole or between university subjects. This preference may reflect a desire to increase the potential recruitment market or to reduce administrative costs, or may reflect a philosophical commitment to freer student choice or greater student autonomy. Should a general move away from prerequisites really be cause for concern?

From the perspective of Subcommittee (i), charged with considering school mathematics, the absence of school mathematics subject prerequisites for university entry makes it more likely that

– students will avoid school mathematics subjects, thinking that other subjects lead to higher ATAR[10];

– schools will abandon the teaching of intermediate or advanced mathematics subjects in favour of elementary subjects (as student demand weakens and intermediate or advanced mathematics are "not essential for tertiary entry" even for STEM degrees).

That enrolments in intermediate and advanced mathematics are falling is an established fact.[11] It has not been demonstrated definitively that removal or weakening of university prerequisites has been a major contributor to this (it is hard to see how a definitive proof could be given), but there is considerable anecdotal evidence for this and there is a widespread belief in university mathematical sciences departments that this is the case. Moreover, there is the following evidence that changing the ATAR-attractiveness of year 12 intermediate or advanced mathematics study does increase the likelihood that students will select these subjects. A University of Queensland selection bonus for students with Mathematics C (the highest Queensland year 12 mathematics subject) was announced to apply for 2011 entry and therefore could have affected year 12 mathematics enrolment patterns from 2010 onwards. See Table 6 for the consequent recovery in Mathematics C enrolments.

The preceding discussion has been in the context of university prerequisites as a driver for school enrolments, but one must ask *"Are prerequisites needed for tertiary mathematics teaching?"*. There are three major concerns with a decline in the level of school mathematics taken by incoming students.
(i) *Pressure on standards of University subjects.* No department can afford to fail too many students in any of its subjects. A weaker intake can be expected to lead to one or more of the following.

– Reducing achievement levels needed to pass the subject (crudely, accepting lower raw marks).

– Reducing the content or intellectual level of the subject.

[10]Concerning the existence of this belief, see Tovey (2013).
[11]Barrington and Brown (2005), Forgasz (2006), Barrington (2012).

Table 6: Total enrolments in Queensland year 12 mathematics subjects 2004–2013. The change in University of Queensland selection practices (granting a bonus for Mathematics C that affected student year 12 subject choice in 2010 and beyond) made students who took Mathematics B somewhat more likely to take Mathematics C also.

Raw data for 2013 provided by Darinka Copak, Senior Data Management Officer, Data Management Unit, Queensland Studies Authority. Data for earlier years from http://www.qsa.qld.edu.au.

Year 12 group	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Maths A	21247	21565	22015	23102	23616	24432	25327	25910	26533	26513
Maths B	16317	16535	15989	16270	16118	16434	16598	16838	17052	17388
Maths C	3430	3317	3226	3343	3397	3550	3876	4000	4182	4564
Ratio B/A	0.768	0.767	0.726	0.704	0.683	0.673	0.655	0.650	0.643	0.656
Ratio C/B	0.210	0.201	0.202	0.205	0.211	0.216	0.234	0.238	0.245	0.262

– Redirection of resources to support the subject (increased expense per student).

– Creating either bridging subjects taught in standard ways or intensive bridging courses which may not allow sufficient time for conceptual development and consolidation of learning.

– Losing the subject altogether (especially if the subject is offered as a service subject).

(ii) *Transfer of resource demands.* It is hard to see repairing school skills at university through additional bridging subjects, or providing greatly increased levels of support per student, as cost-effective and there is no evidence that resources will flow to departments, or to individual staff, who have to deal with the problem of weaker students.

(iii) *Reduced standards at graduation.* When prerequisites for degree entry exist, applicants who lack those prerequisites have to make them up before commencing the degree. For them, background repair does not erode content within the degree itself. Without prerequisites, bridging subjects are likely to be taken for credit as part of the degree, reducing the total number of university-level subjects that can be accommodated within a degree of fixed length. It is particularly troubling that even some of the stronger universities, disciplines with strong quantitative and modelling aspects can appear to require nothing from school in terms of mathematics. For example, the Macquarie website lists a generic BSc[12] and a BSc specialising in Climate Science[13] with neither prerequisites nor assumed knowledge of mathematics or any science.

A fascinating 2007 study[14] of the effect of choice of high school subjects on subsequent success in undergraduate science studies concluded that "The two pillars supporting college science appear to be study in the same science subject and more advanced study of mathematics in high school." Tellingly, the authors reported that "Years of mathematics instruction (ymi) was a significant predictor of performance across all college science subjects, including introductory college biology, a discipline not traditionally associated with strong mathematics preparation" but none of the three traditional high school subjects Biology, Chemistry of Physics had such beneficial effects on study of ether of the other two traditional subjects at undergraduate level. A recent study at the University of Wollongong confirmed the importance for success in undergraduate chemistry of senior secondary school mathematics study to at least intermediate level.[15]

[12] http://courses.mq.edu.au/undergraduate/degree/bachelor-of-science (seen November 2013).

[13] http://courses.mq.edu.au/undergraduate/degree/bachelor-of-science/major-in-climate-science (seen November 2013).

[14] Sadler & Tai [19]; P. Hall, "Comparing four pillars of wisdom", http://www.thefunneled-web.com (22 August 2007).

[15] Reported by Dr Glennys O'Brien, Director of First Year Studies, School of Chemistry, University of Wollongong, during the *National Forum on Assumed Knowledge in Maths: its Broad Impact on Tertiary STEM Programs*, University of Sydney, 13–14 February 2014. For some details of the study, a student project, see http://eis.uow.edu.au/content/groups/public/@web/@inf/@math/documents/doc/uow127144.pdf and https://eis.uow.edu.au/content/groups/public/@web/@inf/@math/documents/doc/uow122902.pdf and the preprint [1]. This study reinforces conclusions from a broader study at the University of Western Sydney [18]: "The performance of first-year students in four different mathematics and mathematically related subjects is compared to the level of their secondary school mathematics and performance, and to their tertiary entrance score. We conclude that a student's secondary school mathematics background, not their tertiary entrance score, has a dramatic effect on pass rates."

In the recent House of Lords review of Higher Education in Science, Technology, Engineering and Mathematics (STEM) subjects the U.K. government was advised[16] to make studying mathematics in some form compulsory for all students continuing at school beyond age 16, and to make mathematics to A2 level a requirement for students intending to study STEM subjects in higher education.

> **FINDING 2** The subcommittee finds that the general move away from enforced prerequisites for entry to undergraduate degrees, or for entry into subjects within degrees, threatens the viability of senior secondary school mathematics, transfers work (but not resources) to university departments, and lowers the quality of graduates produced.

The subcommittee believes that the continued decline of intermediate and advanced mathematics relative to elementary mathematics at senior secondary level must be reversed and that an enforced minimum prerequisite of intermediate mathematics (Mathematical Methods in the proposed National Curriculum) is needed for all undergraduate degrees with significant mathematical sciences content. The subcommittee notes evidence that better school mathematics preparation has positive benefits for undergraduate study in all STEM areas. Put simply, *to fix STEM problems, start by fixing school mathematics problems.*

The likelihood that schools will offer advanced mathematics may reasonably be expected to increase as the numbers taking intermediate mathematics grows. It may be possible to sustain or expand niche undergraduate programs that require advanced mathematics at school, but there seems little hope of recovering advanced mathematics as a strict prerequisite for larger-intake programs such as the Bachelor of Engineering in the short term. For the three states studied, the entry requirements of the institutions that compete most aggressively for high ATAR students tend to align, and fears of losing market share or cutting equity group intake may make unilateral moves to raise standards unpopular with executive management, no matter how compelling the academic case.[17]

> **RECOMMENDATION 1** (*addressed to universities and to the professional societies that accredit the relevant degrees or majors within them*) With an appropriate period of notice to stakeholders, year 12 intermediate mathematics should be set as a degree entry prerequisite for Bachelor of Science, Bachelor of Engineering and Bachelor of Commerce programs. To cater for equity groups unable to access intermediate mathematics at school, pathways that extend degrees should be created to allow students to acquire necessary prerequisite material before commencing normal degree studies.

The subcommittee notes that in his review of some mathematical sciences issues for the Group of Eight universities [7], Professor Gavin Brown recommended collaboration between Go8 universities to "develop a systematic structure of enabling programs to counter the drop in students entering with low mathematics experience", noting that this proposal "has a strong equity dimension and should include careful lobbying for government support". In its submission to the Decadal Plan,[18] the Australian Mathematical Sciences Institute concurs with Professor Brown: "... study pathways for students, mature-aged and otherwise, need to cater for those without adequate preparation for university study in the discipline. A collaborative approach to the delivery of the subjects required is highly desirable and, with the advent of the national curriculum, more achievable than before."

While endorsing efforts to support present students disenfranchised by the inaccessibility of appropriate school mathematics or by poor guidance in subject selection, the subcommittee does not want such efforts to be seen as an acceptable alternative to the more challenging, but in the long term much more satisfactory, campaign to reverse the decline in school mathematics.

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[16] House of Lords [10], paragraph 254, Recommendation 2

[17] Generally, initiatives seem to involve reductions in prerequisites, which competing institutions move to match. For example, when QUT dropped advanced mathematics (Maths C) as a prerequisite for Engineering, UQ soon followed.

[18] The submission (dated 31 May 2013)) is relevant to all four of our themes: see http://www.mathscidecadalplan.org.au/submission-by-amsi-management-committee-2-mathematics-and-statistics-education-and-training-in-universities/.

Table 7: Draft Threshold Learning Outcomes for undergraduate degrees with a major in mathematics and/or statistics, proposed by the Australian Mathematical Sciences Learning and Teaching Network .

TLO1	Mathematical thinking	Understanding the ways of thinking in the mathematical sciences including different approaches in different areas
TLO2	Discovery and problem solving	Investigating and solving straightforward problems using mathematical and/or statistical methods
TLO3	Communication	Communicate mathematical and statistical information, arguments, or results for a range of purposes using a variety of means
TLO4	Responsibility	Demonstrate personal, professional and social responsibility

RECOMMENDATION 2 (*addressed to universities and to the state tertiary admissions centres*) To encourage capable students to enrol in advanced mathematics at school, in each state and territory either or both of the following arrangements should be in place:
(a) in the computation of the ATAR, subject results are scaled to ensure that students who undertake more challenging subjects populated by more able students are not disadvantaged relative to students of similar ability who choose easier subjects with less competition;
(b) selection bonuses are awarded to students with good results in advanced mathematics who apply for degrees with mathematical sciences content.

1.2 *Are pathways directly into employment covered appropriately?*

Other subcommittees for the Decadal Plan have considered the employment of graduate mathematical scientists so we comment only briefly on this here. There is a general consensus that insufficiently many good mathematical sciences graduates are currently being produced. In his submission to the Decadal Plan, John Henstridge (Managing Director, Data Analysis Australia) makes the following observation.[19] "One impact I observe is that it is increasingly difficult to recruit appropriate staff within Australia. Quite simply, Australia is not producing enough mathematically trained people at all levels. Data Analysis Australia is now reliant upon recruiting internationally—currently New Zealand is our primary resource for recruits at all levels. This observation is consistent with the evidence that suggests that compared with comparable countries—New Zealand, the United Kingdom, Canada and the USA—Australia is graduating only 40% the number of mathematical scientists."

Certain specialisations within the mathematical sciences are more overtly vocational (statistics, operations research, scientific computation) but the subcommittee does not propose any coercion concerning specialisations to be offered, believing instead that provided the level of mathematical sciences education and general skills developed by students are appropriate, a variety of specific mathematical sciences backgrounds will lead to useful and rewarding employment. In support of this broad view, we note a submission from the Royal College of Pathologists of Australasia, that highlights a number of areas including some that are traditionally viewed as pure mathematics.[20] "The College would like to recommend that the mathematics curriculum include pertinent biological and medical examples so that students realise the potential to apply it to pathology and medicine. Examples might include:
– Biostatistics (e.g . reference intervals of large datasets, microarrays)
– Probability (e.g. Bayesian calculations, QC and process control)
– Graph theory (e.g. Massively Parallel Sequencing and sequence assembly)
– Inferential Logic (e.g. Rules-based smart requesting and reporting)
– Analytical Maths and Geometry (e.g. Machine Learning, Support vectors, Component Analysis, Visual displays)
– Computer Science (e.g. health databases, genomic comparison algorithms, etc)."

[19] http://www.mathscidecadalplan.org.au/602/ (16 May 2013); a typographical error in the web page text is corrected here.
[20] http://www.mathscidecadalplan.org.au/submission-from-the-royal-college-of-pathologists-of-australasia (18 May 2013).

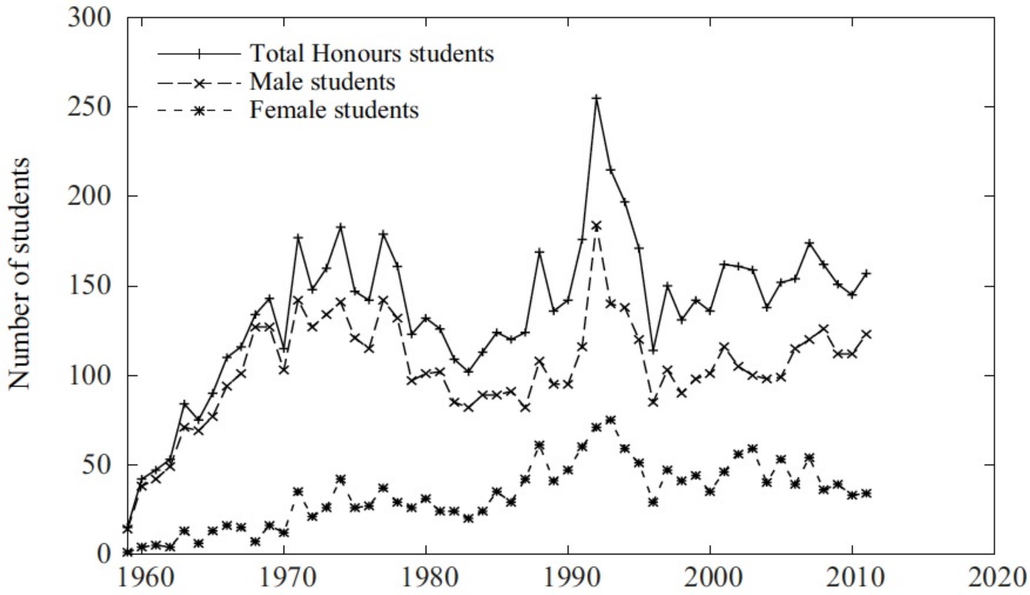

Figure 1: Graduations from RHD pathway programs in mathematics and statistics up to 2012. Data is for honours and the MSc at the University of Melbourne, which has replaced its honours program; reproduced from [3] with permission from the Australian Mathematical Sciences Institute).

The Australian Mathematical Sciences Learning and Teaching Network (AMSLaTNet) has developed draft "Threshold Learning Outcomes" (TLOs) for undergraduate mathematical science majors,[21] which are summarised in Table 7. The subcommittee notes that programs that conform to these aspirations are likely to produce highly employable graduates.

1.3 *Are pathways to further study, including research higher degrees, covered appropriately?*

In recent years Australia has produced around 150 graduates per annum in "RHD pathway" programs, which are Bachelor Honours or 'coursework and project' based MSc degrees that are the gateway to research higher degrees (MSc by Research, M. Phil., or Ph.D.): see Fig. 1. The best available data[22] has been assembled over many years by Griffith University mathematician Peter Johnston, sourced from university departments by request. Although there are occasional gaps for individual institutions that fail to report in a particular year, the data is highly informative. There is a serious gender imbalance that needs to be rectified (an issue taken up under Theme 4 later in this report). An additional particularly worrisome feature of the data is the concentration of RHD pathway graduations in a small number of institutions. To avoid misleading conclusions when an institution fails to report occasionally, it is helpful to consider the largest number of graduates and the smallest number of graduates that an institution produced in any one year over a number of years. In Table 8, we show data based on degree completions for the years 2010–2012. The following observations need to be made.

(a) On a given campus, year-to-year fluctuations in numbers of students in these key "pathway to research higher degree" can be large.

(b) Since students commencing research higher degrees often do so at the universities from which they graduated from RHD pathway programs, many universities face challenges sourcing research students.

(c) The supply of potential research higher degree students is strongly dominated by a small group of universities in which the Group of Eight (Go8) universities dominate. Of the non-Go8 universities with a strong output, only one (the University of Wollongong) is located outside of a capital city.

(d) The output of potential research higher degree students from regional and rural universities is tiny.

[21] http://www.acds.edu.au/tlcentre/wp-content/uploads/2013/05/Mathematical-Sciences-Draft-TLOs-and-Development.doc

[22] For the most recent three years of data, see Johnston [11, 12, 13], in which references to earlier data are also given.

Table 8: Largest graduation from RHD pathway (Honours and 'coursework and project' MSc) in any one year in the period 2010–2012, based on Johnston's data [11, 12, 13]. For those institutions with no reported graduations in precisely one of the three years, we conjecture a data gap and report the smaller of the two nonzero values as the minimum.

"Group of Eight" universities	minimum	maximum
Australian National University	9	11
Monash University	15	23
University of Adelaide	4	11
University of Melbourne	13	25
University of New South Wales	7	10
University of Queensland	10	24
University of Sydney	18	27
University of Western Australia	3	8
"Australian Technology Network"	**minimum**	**maximum**
Curtin University of Technology	0	3
Queensland University of Technology	4	8
RMIT University	2	4
University of South Australia	6	6
University of Technology Sydney	3	6
Other universities in major urban areas	**minimum**	**maximum**
Australian Catholic University	0	0
Australian Defence Force Academy (UNSW@Canberra)	0	1
Bond University	0	0
Deakin University	1	2
Edith Cowan University	0	1
Flinders University	1	5
Griffith University	1	1
Latrobe University	7	10
Macquarie University	2	3
Murdoch University	3	3
University of Canberra	0	0
University of Newcastle	1	5
University of Tasmania	4	7
University of Western Sydney	0	1
University of Wollongong	4	14
Victoria University	0	0
Regional and rural universities	**minimum**	**maximum**
Charles Darwin University	0	1
Central Queensland University	0	0
Charles Sturt University	0	0
James Cook University	0	0
Southern Cross University	0	0
University of Ballarat	0	2
University of New England	0	1
University of Southern Queensland	0	1

The subcommittee also notes that the absence of an RHD pathway on a campus makes the existence of a mathematical sciences major on that campus less secure. The challenge of offering a viable pathway program with limited staff and small student cohorts is taken up under Theme 3.

FINDING 3 The subcommittee finds that the concentration in a few universities of pathways preparing students to enter research higher degrees in mathematics and statistics is problematic, especially outside capital cities.

1.4 *How do the amount of study (however measured), breadth of study and depth of study compare to relevant international models?*

Finding suitable international comparators is not entirely straightforward. It is important to recognise two fundamentally different approaches to disciplines within undergraduate degrees, which we illustrate with the U.K. and U.S. models.

In the U.K.,[23] students aspiring to study mathematics as undergraduates typically apply for a specific mathematics stream, such as B.Sc. (Mathematics), rather than entering a broad degree. Reflecting this, the three-year undergraduate degree consists overwhelmingly of mathematics and statistics subjects, with the modest number of other subjects (if any at all) usually drawn from cognate disciplines, and combined programs between mathematics and one of physics or computer science can consist of nothing but mathematics and its companion discipline. In the U.K. undergraduates can extend their degree by one year to take out a Masters degree. This may require less project work than the typical minor thesis requirements found in Australian fourth-year honours programs or coursework-based masters programs, but the total amount of coursework in the mathematical sciences that is prescribed in the U.K. model (exiting at Bachelor of at Masters level) is typically greater than the maximum accessible in the Australian model, and much greater than the Australian minimum requirements to exit with an undergraduate mathematical sciences major, honours degree or coursework masters degree.

It is typical in the U.S. (as it is in Australia) for students aspiring to major studies in the mathematical sciences apply for a broad degree. It is not possible for the student to focus exclusively on mathematics. At first year level in Australia, it is rare for students to take even as many as four (of the customary eight subjects per year) mathematical sciences subjects, with two or three being more common and U.S. universities expect students to sample various disciplines in the freshman year. The enforced spread of interests, a feature of the underlying philosophy of the American liberal arts education model, may be advantageous in preparing students for future interdisciplinary work and is not at variance with proper mathematical training, provided that mathematical sciences subjects in later years are sufficiently abundant to enable a stronger focus on mathematics and statistics. The amount of mathematical sciences material typically required to meet minimum requirements for a major in a three year Australian undergraduate degree appears broadly comparable to but perhaps slightly less than what is required in a four-year North American undergraduate degree, but the longer North American model does accommodate the possibility of more total mathematical exposure for students seeking to meet more than minimal requirements for a major.[24]

European models are also worth considering, in view of the popularly perceived relevance of the so-called "Bologna model" to the evolution of Australian higher education. The Bergen Conference of European Ministers Responsible for Higher Education 19-20 May 2005 adopted a three-cycle framework for tertiary education, with the first cycle, corresponding essentially to Bachelor degrees, with these

[23]For the requirements for mathematical sciences majors at Cambridge and Oxford, and some combined programs, see http://www.study.cam.ac.uk/undergraduate/courses/maths/ and http://www.ox.ac.uk/admissions/undergraduate_courses/courses/mathematics/mathematics_course.html.

[24]In the US and Canadian systems typical major requirements appear to be 12 to 13 subjects at around the typical size of an Australian subject, though many students will take more. The requirements may be stated as a given number of preparatory subjects and a given number of upper-level subjects. The 12 or 13 noted here covers both contributions and several preparatory subjects may be at Australian year 12 standard. Although it has been folklore for some time the U.S. school students do no meet calculus, suggesting that U.S. initial university subjects might be at a lower level, it is now common for U.S. school students with interests in and aptitude for mathematics to take advanced placement subjects including calculus while at school.

attributes.[25] "Qualifications that signify completion of the first cycle are awarded to students who:

– have demonstrated knowledge and understanding in a field of study that builds upon their general secondary education, and is typically at a level that, whilst supported by advanced textbooks, includes some aspects that will be informed by knowledge of the forefront of their field of study;

– can apply their knowledge and understanding in a manner that indicates a professional approach to their work or vocation, and have competences typically demonstrated through devising and sustaining arguments and solving problems within their field of study;

– have the ability to gather and interpret relevant data (usually within their field of study) to inform judgments that include reflection on relevant social, scientific or ethical issues; – can communicate information, ideas, problems and solutions to both specialist and non-specialist audiences;

– have developed those learning skills that are necessary for them to continue to undertake further study with a high degree of autonomy."

It would be interesting for Australian mathematical sciences departments to consider whether students following their present major sequences would fulfil all of these requirements.

The Australian undergraduate model may be a little light in content compared to European Bologna system model in terms of the total amount of study undertaken per year. The work requirements of the Bologna model are quantified in terms of the European Credit and Transfer System (ECTS), with one ECTS credit equivalent to 30 hours of study (class time plus private study). A typical yearly load is 60 ECTS credits, corresponding to 1800 hours of study. With two 12 or 13 week semesters and some allowance for work during mid-semester breaks and between the end of classes and final examinations, an Australian undergraduate is committed for the equivalent of 30 weeks per year. The rather optimistic assumption that students spend 40 hours per week on studies (questionable given the amount of paid part-time work that many students undertake) only produces around 1200 hours of study per year.

1.5 *Are there particular topics that are essential, either for all mathematics and statistics, or at a finer level, e.g. for all pure mathematics majors or for all statistics majors?*

The subcommittee does not favour being prescriptive as to the content of Mathematical Sciences undergraduate majors, and also recognises that degree rules covering degrees within which majors sit may prevent aspirational proposals on total mathematical sciences content being realistic. The subcommittee does have recommendations on broad attributes of Mathematical Sciences undergraduate majors.

> **RECOMMENDATION 3** (*addressed to universities*) Every undergraduate mathematical sciences major should
> (a) have at least 50% of final-year credit devoted to mathematical sciences subjects;
> (b) have both depth in one specialised area of mathematical sciences and reasonable breadth across areas of the mathematical sciences;
> (c) develop the student's understanding of the context and applications of the mathematical sciences;
> (d) develop the student's ability to communicate mathematical or statistical concepts or results to both specialists and lay people;
> (e) develop facility with appropriate information technology and professional software.

The subcommittee does have some concerns on the type of teaching and assessment that occurs and on the associated learning strategies of undergraduate students. We draw a distinction between a learning style based purely on rote learning and developing facility with algorithmic processes and one based on developing insight and understanding. The Australian Government's Office for Learning and Teaching has funded a multi-university project entitled "Evidence based resources for overcoming algebraic misconceptions that inhibit students' progress in tertiary mathematical sciences"[26] This project raises

[25] http://www.ehea.info/Uploads/QF/Bologna_Framework_and_Certification_revised_29_02_08.pdf.

[26] University of Melbourne 2012–14: http://www.olt.gov.au/system/files/2012_Round2_Grants_Announcement_v1-1.pdf.

important issues, two of which may be usefully noted here.[27]

“Our conjecture is that some students ‘slip’ through the system by practicing answering standard examination questions but still holding incorrect and incomplete mathematical conceptions. The student data collected . . . supports this conjecture.”

“. . . Correctly targeting misconceptions and giving the best advice for teaching and learning (often using very carefully chosen examples that expose students’ thinking and create cognitive conflict OR a physical or graphical representation) is key to success. . . ”

The subcommittee does not feel it appropriate at this stage to make concrete suggestions as to how the development of conceptual understanding can be enhanced. Given that in terms of total exposure to mathematics topics we are a little behind the best international models, raising the quality of the learning by students of the material that we are able to present and the nurturing of generic mathematical skills is especially important.

1.6 *What is the role of technology as a teaching tool, and in hands-on student learning?*

A distinction needs to be drawn between the use of technology to enhance learning in an on-campus experience including face-to-face teaching, and the use of technology to reduce or totally replace face-to-face teaching. The latter strategy is sometimes advocated as a means either to reduce the cost of teaching the students we have, or of attracting additional (revenue generating) students who could not be accommodated in campus-based teaching, but a number of possible reservations need to be raised.

- There is a significant development cost for online teaching (rather than simply placing print material on an internet site).
- Allowing for staff time in development and running, individual on-line feedback to students if offered and assessment grading, the savings for local teaching and revenue from external students generated by going on-line with in-house developed material may be inadequate unless the class sizes are high and the amount of individual contact with or feedback to students is low.[28]
- There will be competition for students seeking online subjects or whole courses from higher-ranked international competitors who have already invested heavily in this area.
- Ensuring integrity in assessment is problematic for purely online subjects.

Very recently there has been an explosion of interest in Massive Open Online Courses (MOOCs),[29] which raise new challenges. There may be pressure to accept MOOCs to meet formal prerequisites or supply assumed knowledge, despite there being either no formal assessment or only assessment that is not quality assured. There may also be pressure to accept MOOCs for credit towards courses, eroding local teaching income.

Turning to the use of technology to enhance more conventional teaching, we see opportunities for innovation, although funding the supply of both hardware and software may be challenging. Exposure to one or more of the standard professional software packages used by working mathematical scientists[30] has been advocated under Recommendation 3 and is important preparation for future careers. These packages can also be used to enliven teaching and enable more realistic or more challenging applications to be addressed. The subcommittee does not see it as appropriate to make detailed proposals for the use of specific software, or for the use of software in particular ways or for particular topics.

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[27] These observations by project participant Assoc. Prof. Robyn Pierce, were provided to the subcommittee by Dr Deb King.

[28] Semester Online (semesteronline.org) promises simultaneous on-line experiences for small groups (15–20 students) and the development work to produce a subject to the premium presentations standards that they espouse is very substantial.

[29] Links to available courses can be found through a variety of internet portals, such as http://www.mooc-list.com. Major providers of MOOCs or of other free on-line educational materials include Coursera, edX, NovoED, and iTunesU.

[30] We mention as examples in no particular order the commercial packages Mathematica, Matlab and Maple, and the freely available R and Octave, but there are many other useful packages available.

Theme 2. Service Teaching

The subcommittee recognises that the mission of mathematical sciences departments is broad, and that to be successful departments need to pay due attention to all of their key responsibilities. Under Theme 1 we have addressed the training of undergraduate mathematics and statistics majors, some of whom will proceed to further mathematical sciences study (including pathways to research higher degrees). Other undergraduate teaching may be described broadly as "service teaching". The term "service teaching" can be narrowly interpreted as teaching across faculty boundaries (for example, a Faculty of Science based mathematics department teaching subjects to students enrolled in Engineering Faculty degrees), but we prefer to conceive of all teaching of mathematics and statistics to students whose main interests lie elsewhere (and who may have no interest in mathematics per se) as service teaching.

There is a fundamental principle to which the subcommittee subscribes that material strongly associated with a discipline should be taught by, or in collaboration with, the discipline experts. This principle can be defended on philosophical grounds, based on a particular perspective on the modern, comprehensive university, or on educational grounds, but there are pragmatic considerations that must be discussed openly. Under present funding arrangements, no mathematical sciences department can survive solely on the income generated from its own majors and it is unrealistic to expect this to change. Over the period 2002–2010, total EFTSL[31] in mathematical sciences subjects offered to students enrolled in science undergraduate degrees or in combined undergraduate degree programs involving a science degree remained relatively stable at around 12,000 (a modest decline from 2003 to 2006 has been reversed).[32] Only biological sciences carry a larger EFTSL load than mathematical sciences in science degrees. Comprehensive figures on service teaching beyond faculty boundaries appears harder to obtain. Among AMSI members,[33] over 95% provide service teaching to engineering, but only about half of the AMSI members departments currently provide service teaching for each of the broad groupings "health science and nursing", "business and accounting" and "economics and finance".

2.1 *How do we ensure that mathematics and statistics departments have an ongoing role in the teaching of mathematical and statistical material to students who are not mathematics or statistics majors?*

It has long been recognised that there can be financial advantages to other faculties in teaching mathematics and statistics in-house, rather than using the services of the local mathematical sciences discipline, and that financial preoccupations can over-ride educational imperatives.[34] Prudent high-level oversight and clear policies are needed if these financial pressures are to be overcome. However tensions over service teaching issues are not always solely over cost. There can be significant practical expertise in narrow mathematical areas (often to a high level of sophistication) residing in staff outside mathematical sciences departments, and in some cases, staff with excellent mathematics credentials are employed in other departments. Proposals to teach discipline-specific useful mathematics and statistics efficiently, in context and with minimal class time, can be seem as very attractive alternatives to contracted service teaching.

A report arising from a 2008 Carrick Institute study [6] made the following important observation. "In order to accommodate topics in professional practice, the number of mathematics subjects has been reduced. This has necessitated the removal of some mathematics topics from the compulsory part of the curriculum but there is widespread disagreement on which topics should have the lowest priority. Compared to 20 years ago, it is now less likely for a 4-year BE graduate to be extensively trained in mathematics. The niche for mathematically strong engineers is being populated by a relatively small

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[31]EFTSL = effective full-time student load.

[32]See [17] p. 84, Figure 4.4.16; also reproduced in the AMSI *Discipline Profile of the Mathematical Sciences 2013* [3].

[33]See the AMSI *Discipline Profile of the Mathematical Sciences 2013* [3].

[34]In the foreword to the report of an ALTC study on quantitative skills in science [15], John Rice (Executive Director, Australian Council of Deans of Science) wrote "…the study points out what most people already suspect, that curriculum redesign involving quantitative skills is often regarded as driven by the politics of student load redistribution and/or 'teaching efficiencies', inhibiting serious consideration of the significant educational issues."

number of double degree students. These students are important as they make a significant contribution to Australia's mathematical capability."

Tensions over service teaching issues may be lessened in cases where there are joint appointments shared between the teaching department and the department being serviced, or where interdisciplinary collaborations between the departments exist. There need not be a conflict between the aspiration to promote interdisciplinary work and the need to sustain the fundamental disciplines.

Mathematics sciences departments wishing to provide service teaching need to be able to do it very well. A mutually satisfactory service teaching arrangement requires good faith from all parties, realistic syllabus design and realistic expectations on student performance and the level of service provided, given the available class time and the fee for service.

> **RECOMMENDATION 4** (*addressed to university mathematical sciences departments and to universities more broadly*) Mathematical sciences departments should offer an appropriate sequence of subjects for each of the following undergraduate cohorts:
>
> – students seeking mathematical sciences knowledge to assist them in pursuing other majors or other degrees ("service teaching");
> – students seeking to develop perspectives and skills for employment on graduation ("mathematics or statistics as a mindset");
> – students seeking mathematical sciences careers, who need pathways to postgraduate coursework study and to research higher degrees ("mathematics or statistics as a profession").

Departments should ensure that necessary resources are allocated to each of these cohorts, that each cohort is valued, nurtured and taught with enthusiasm, and that staff are recognized and rewarded for contributions to the proper education of any of these cohorts.

The subcommittee observes that mathematics and statistics play an increasingly large role in science, technology, medicine and commerce. Active involvement of mathematical sciences departments in teaching mathematics and statistics to these and other disciplines is important for the development of the disciplines, and for the long-term viability of mathematical sciences departments themselves. The subcommittee notes that implementing the preceding recommendations will assist mathematical sciences departments in gaining or retaining service teaching, but adds an additional recommendation.

> **RECOMMENDATION 5** (*addressed to Universities Australia and individual universities*) Universities should ensure in teaching of mathematical sciences material to undergraduate students not undertaking mathematical sciences majors, the mathematical science discipline plays a major role. In arranging for such service teaching, there should be active good-faith dialogue and collaboration to ensure the compatibility of the syllabus, standards set and pass rates, and resources available. Mathematical sciences departments allocated service teaching must deliver services and outcomes commensurate with reasonable expectations.

2.2 *What are the roles of more generic subjects vs highly customised subjects for specific service teaching client groups?*

The subcommittee has considered this issue, but has the view that there is no "one size fits all" answer here. More generic subjects, if well taught, may impart more broadly transferrable mathematical skills, and subjects that address mathematics more deeply, if well taught, can lead to better understanding, but the specific subject material and skills required by some disciplines can make it impractical for their students to share classes with students with very different interests and needs.

Theme 3. Collaboration and Cooperation

The subcommittee notes that another subcommittee for the Decadal Plan [Subcommittee (i)] carries responsibility for addressing the training of future school teachers, but records here its view that active collaboration between mathematical sciences departments and faculties of education or graduate schools of education is highly to be desired.[35]

Subcommittee (iii), which addresses mathematics and statistics research, may address issues of cooperation in the provision of research seminars and advanced coursework for research higher degree students and we do not take up this issue. We do note, however, that mechanisms for such cooperation may be able to be integrated with arrangements for collaborative resourcing of honours and coursework postgraduate degree programs, on which we comment below.

3.1 *In seeking to maintain a viable national mathematics and statistics community, to what extent should there be shared or cooperative teaching arrangements?*

3.2 *How are such arrangements best implemented?*

The subcommittee recognises that some universities will be unable to offer a variety of undergraduate majors across the mathematical sciences (for example, majors in "pure mathematics", "applied mathematics" and "statistics"), but it is a reasonable expectation that each university should offer at least one undergraduate major in the mathematical sciences, from which entry to post-Bachelor study (postgraduate diploma, honours year or coursework master degree) is possible. Post-Bachelor study might in most cases be in pathways that lead ultimately to research higher degrees, but could also include specialist pathways to employment based on statistics, operations research, mathematical finance, or other mathematical science areas

Departments unable to mount post-Bachelor programs on their own should either establish pathways for their students to continue study at other Australian universities, or participate with other Australian universities in shared post-Bachelor programs. Cooperative arrangements could also enable post-Bachelor students at all Australian universities to have access to a wider variety of subjects than can be supported at a single institution.

> **RECOMMENDATION 6** (*addressed to universities*) Universities should to ensure that all Australian students completing undergraduate mathematical sciences majors have pathways to subsequent higher-level study. Where individual universities are unable on their own to support such pathways, and to enhance the diversity of higher-level study that is available, universities should cooperate in the provision of pathways to further study.

It may be possible for capstone subjects in higher-level programs to be developed in collaboration with AMSI or other stakeholders. Opportunities for cooperation within undergraduate programs are also worthy of consideration, especially in service teaching, in the context of the foreshadowed national senior secondary school curriculum, and given the urgency in providing upskilling for school mathematics teachers. Cooperative activities with a variety of stakeholders are also likely to be helpful in developing the curriculum (both at undergraduate level and at higher levels) for modern applications and in providing ongoing professional development opportunities for graduates.

The subcommittee does not feel that it is appropriate to be prescriptive about how shared programs should be presented. Online methodologies will naturally be considered, but there may be more economical or convenient alternatives to the current Access Grid Room arrangements, which have substantial capital and staffing costs. The Skype package has changed the way mathematical scientists collaborate in pairs or small groups, and inexpensive, easy to maintain protocols for shared teaching may emerge in the near future.

[35] In a 2009 report to the Vice Chancellors of Group of 8 universities [7], Gavin Brown made the following formal recommendation. "The Go8 should encourage dialogue between Faculties of Education and Mathematics Departments with a view to introducing a component in the primary training program giving mathematical confidence and resources to future teachers. This would be taught by the Mathematics Department or School." It should be added that exemplars of existing successful collaborations should be sought, and that the selection of appropriate staff to lead such programs requires very careful attention.

Table 9: Demographic distribution and spread between year levels of effective full-time student load in mathematical sciences subjects in 2012 for AMSI member universities [3].

Demographics	Go8			non-Go8		
Domestic male	56.02%			53.19%		
Domestic female	26.25%			30.25%		
International male	11.34%			9.33%		
International female	6.39%			7.23%		
Year level distribution	**Go8**			**non-Go8**		
year level	1st	2nd	3rd	1st	2nd	3rd
EFTSL	4316	1976	687	2651	1399	480
relative to 1st year		45.8%	15.9%		52.8%	18.1%
relative to 2nd year			34.7%			34.3%

Theme 4. Diversity in student recruitment and retention

The most recent snapshot of aggregated undergraduate enrolment in mathematical sciences appears to be the 2011 and 2012 data for AMSI member universities [3]. The 2012 data is presented in Table 9. The 2012 total effective full-time student load (EFTSL) at Go8 universities exceeds that at the more numerous non-Go8 universities at all year levels. Since non-Go8 universities typically have greater enrolments from low-SES, rural and indigenous students, this suggests limited take-up by those students of mathematical sciences subjects. Counting individual students rather than EFTSL, around 38,000 individual students took one or more undergraduate mathematics subjects. Two characteristic features of the data in Table 9 are immediately apparent.

– *Undergraduate enrolments are strongly gender imbalanced* (especially in the Go8 universities). This explains the severe gender imbalance in pathways to research higher degrees (shown in Fig. 1 on p. 10) and also limits the supply of female work-ready Bachelor degree graduates with mathematical science skills.
– *The effective full-time student load at third-year level is small relative to the first-year load.* Adequate flow-through to third-year level is crucial to supplying enough mathematical science majors for direct employment, articulating prospective school mathematics teachers into postgraduate teaching programs, and sustaining viable postgraduate coursework and research higher degree programs. (The subcommittee was not tasked with estimating how many graduates need to be produced for each of these post-Bachelor career paths, but evidence of insufficient present supply will be found in the reports of other subcommittees.) Moreover, although department income is highly dependent on total student load, there can be financial disincentives applied to departments that teach small-enrolment subjects, even if the total EFTSL taken over all of their subjects is healthy.

The subcommittee originally asked itself two questions under Theme 4.

4.1 *How do we grow numbers of students taking mathematics and statistics, either as majors or as smaller but significant parts of their degrees?*

4.2 *How do we reduce differential attrition and encourage talented students from presently under-represented groups to continue mathematics and statistics studies?*

We now see these questions as closely enough linked that we address them together.

The crucial issue of student choice of school mathematics has already been discussed extensively under Theme 1. This issue has implications not only on the total numbers of students arriving at university well-prepared for study that relies on mathematical knowledge, but also on the demographic distribution of those students.[36] We have already addressed the issue of degree prerequisites in Recommendation 1 (p. 8), but we would like to see more students taking up mathematics and statistics because they see them as attractive in their own right, rather than as necessary evils on the path to a professional qualification.

> **FINDING 4** The subcommittee finds that the correct choice of mathematics subjects at the start of year 11 is crucial for student access to, and subsequent success in, undergraduate programs with significant mathematical sciences content and that raising student and community awareness of this issue is therefore of considerable importance.

> **FINDING 5** The subcommittee finds that there is a need for ongoing publicity to promote the value of undergraduate mathematics and statistics study as a career pathway in its own right and as supportive of many other career choices in modern society.

Mathematical sciences departments, AMSI and other stakeholders should cooperate to provide a steady flow of better, but honest, good news stories about the importance of mathematical sciences and the career opportunities available to mathematical sciences graduates.

> **FINDING 6** The subcommittee finds that the best prospects for increased mathematics and statistics undergraduate enrolments are to be found in attracting and retaining female students and students from less privileged socio-economic backgrounds.

The subcommittee recognises that initiatives that make the mathematical sciences more attractive generally are likely to increase the diversity of students that are attracted, and that well-resourced, well-taught and well-structured programs that students have a realistic possibility of performing well in are more likely to retain students. There are opportunities for staff to modify teaching styles to hold the interest and boost the confidence of students likely to abandon mathematics early in their undergraduate experience [8], and the practices of institutions in other countries that have been successful in recruiting and retaining female students and students from low socio-economic groups are worth studying. Inspirational examples of institutions in the United States that have responded effectively to these challenges can be found in the American Mathematical Society "Exemplary Program Awards"[37] and the American Mathematical Society Committee on the Profession "Mathematics Programs that Make a Difference" awards.[38] The selection criteria for the latter awards bring out the issue very nicely: they seek programs that

1) aim to bring more individuals from underrepresented minority backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to an advanced degree in mathematics, or retain them in the pipeline;
2) have achieved documentable success in doing so; and
3) are replicable models.

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[36]Demographic details of enrolment in mathematics subjects beyond gender or fee type are difficult to extract. Data from the University of Melbourne (students passing complex analysis in 2010–2012) and at the University of Sydney (third year students 2012) seen by the subcommittee and summarised in the table here showed overrepresentation at third year from certain school types. NSW has many more selective government schools than Victoria, but the under-representation in third year of non-selective government schools, which educate most students, is striking. To some extent, this reflects the lower probability that a non-selective government school student will enter a Go8 university. For Melbourne, several schools that are strong feeders to Melbourne generally were underrepresented in third-year mathematics. Attitudes to mathematics and career aspirations acquired by students while at school seem to have a great influence on progression to third-year mathematics.

school sector	Sydney	Melbourne
government nonselective	24.03%	37.93%
government selective	36.43%	6.55%
catholic systemic	5.43%	12.41%
catholic independent	6.98%	1.38%
independent non-catholic	27.13%	41.72%

[37]These awards, offered annually since 2006, are announced in the May edition of the *Notices of the American Mathematical Society.*

[38]See, for example, "2010 Mathematics Programs that Make a Difference", *Notices of the American Mathematical Society* **57** (5) , 650-651 (2010).

In the United States context, criterion 1 refers especially to African-American, Hispanic or Indigenous students, who are quite numerous. Present numbers of Australian indigenous students completing high school with senior secondary mathematics are too few in number to represent potential sources of significant enrolment growth. All reasonable steps to open realistic pathways into mathematics for Australian indigenous students should be considered, although large-scale initiatives that encourage indigenous student retention to year 12 generally may be more beneficial in the long run than mathematics-specific initiatives. If we recognize that ethnic minority group membership in the United States is significantly associated with low-SES status, then the United States examples may be more relevant than they appear on casual consideration. Anything that helps with low-SES recruitment and retention is likely to have spin-off benefits for recruitment and retention more broadly.

On several occasions the AMS "Exemplary Program" awards have recognized universities that have dedicated mathematics centres that provide drop-in assistance, career guidance and evangelisation about the profession, all of which pay dividends. Team projects, outreach and industry liaison activities have also been commended, though presently under-resourced Australian mathematical sciences departments may not be well placed to mount such initiatives, except perhaps at honours or coursework masters level.

Even if the school mathematics preparation of students is strengthened overall in the near future, the challenge of significant diversity in the level of mathematics preparation of incoming students will long remain an issue. Large, well-resourced departments may be able to deal with the diversity of background and prior mathematical achievement of their students by streaming students on entry, but such streaming should not close off pathways to more advanced study. Where streaming is unaffordable or impractical, other solutions to the challenges of cohort diversity need to be sought.

> **RECOMMENDATION 7** (*addressed to universities*) Mathematical sciences departments should develop practical strategies to deal with the diversity of background and prior mathematical achievement of their students and to increase the attractiveness of continuing mathematics and statistics study to all students, but especially to female students.

Closing reflections and additional recommendations

We close our report with some general observations. To provide better teaching and consequent better learning, it is likely for the foreseeable future that mathematics and statistics teaching will remain labour-intensive. Economies gained through large-group lecturing or alternative methods of material presentation have to be balanced against investment in individual attention in one or several of tutorials, laboratory classes, in-office consultations and individualised on-line or email attention. Provision of effective pathways for students with past educational disadvantage or ongoing difficult personal circumstances (important for reasons of equity) entails additional expense. When revenue per student is small, maintaining upper-level undergraduate subjects and RHD pathways becomes difficult. We recall that as a result of the National Strategic Review of Mathematical Sciences Research in Australia [16], which reported in 2006, the federal government increased the relative funding weights for the mathematical sciences, though the extent to which the additional income received by universities for mathematical sciences teaching flowed through to departments was variable.

> **RECOMMENDATION 8** (*addressed to the Australian Government and to universities*) To improve the quality of graduates, improve the proportion of commencing students who graduate and ensure the availability of mathematical science undergraduate majors and pathways to research higher degrees across the nation, adequate per-student funding needs to be available and to reach mathematical sciences departments.

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